MATH 41 MPS / COREQ

COURSE PACK

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Fall 2019

Welcome to Precalculus!

Answer some or all of the following questions in groups of 3 or 4 to get to know the people around you. Write down sa much or as little as your want to about the people you talk with.

1. What name do you go by?

2. What are your education and/or career goals? What motivates you to attend college?

3. What other classes are you taking this quarter?

4. How do you feel about your past experiences with math and math learning? What is your biggest anticipation or concern about Math 41?

5. Do you have any activities or obligations outside of school that you dedicate a lot of time to?

Get the contact information of at least 2 other people in this class.

Chapter 1

Functions

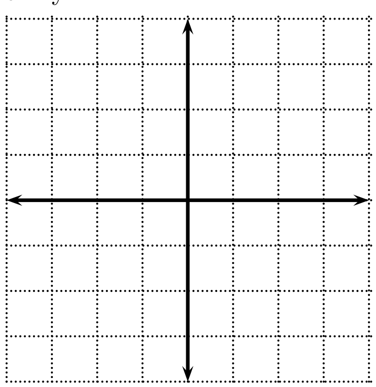
1.1 1.1A: Common parent functions

1. **Identity function**: $f(x) = x$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$

   (c) Domain:

   (d) Range:

   (e) Vertical intercept:

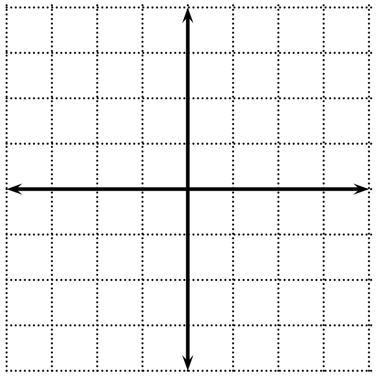
   (f) Horizontal intercept(s):

2. **Absolute value function**: $f(x) = |x|$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Vertical intercept:

   (f) Horizontal intercept(s):

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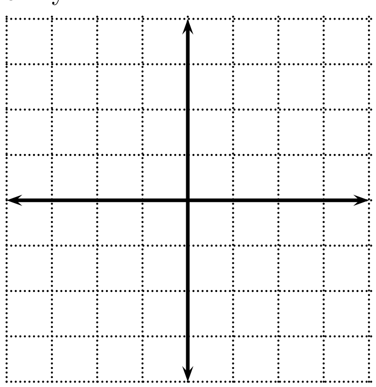
3. **Quadratic function**: $f(x) = x^2$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Vertical intercept:

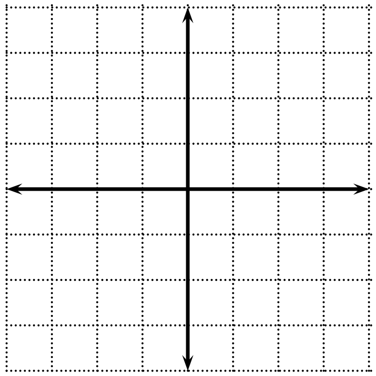
   (f) Horizontal intercept(s):

4. **Cubic function**: $f(x) = x^3$

(a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

(b) Use your table of values to sketch a graph of $f(x)$



(c) Domain:

(d) Range:

(e) Vertical intercept:

(f) Horizontal intercept(s):

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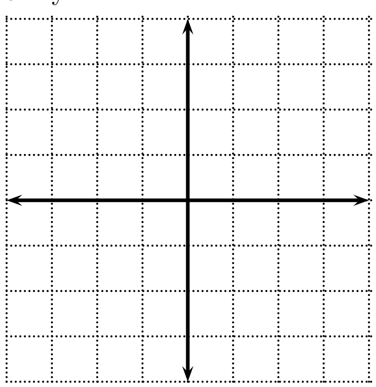
5. **Square root function**: $f(x) = \sqrt{x}$

(a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

(b) Use your table of values to sketch a graph of $f(x)$



(c) Domain:

(d) Range:

(e) Vertical intercept:

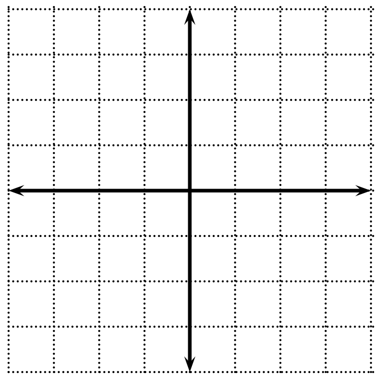
(f) Horizontal intercept(s):

6. **Reciprocal function**: $f(x) = \dfrac{1}{x} = x^{-1}$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Vertical intercept:

   (f) Horizontal intercept(s):

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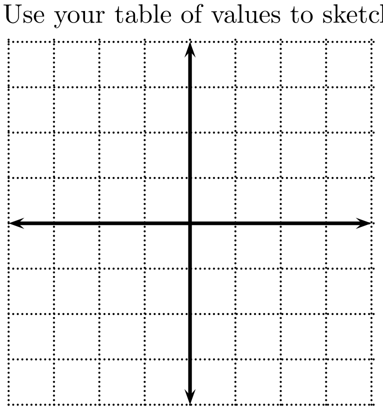
7. **Cube root function**: $f(x) = \sqrt[3]{x}$

   (a) Complete the table of values

x	-3	-2	-1	0	1	2	3
$f(x)$							

   (b) Use your table of values to sketch a graph of $f(x)$



   (c) Domain:

   (d) Range:

   (e) Vertical intercept:

   (f) Horizontal intercept(s):

1.2 1.1B: Functions

Definitions

- Function:

- Domain:

- Range:

- Function notation::

Example 1. *(Try It #2)*

1. Use function notation to express the weight of a pig P in pounds as a function of its age in days d.

2. What does $P(1) = 4$ represent?

Evaluating Equations

Example 2. Find the following for the function $f(x) = 2x^2 + 7$

1. $f(3)$

2. $f(x+2)$

3. $f(3x)$

4. $f(a+h)$

5. $\dfrac{f(4+h) - f(4)}{h}$

Example 3. Find $g(5)$ for $g(x) = \sqrt{x+4}$

Example 4. Find the following for the function $f(x) = 10 - 3x^2$

1. $f(-4)$

2. $f(x-1)$

3. $\dfrac{f(2+h) - f(2)}{h}$

Example 5. Given $g(m) = \sqrt{m+4}$ **solve** $g(m) = 2$.

Example 6. Answer the following given $2n + 6p = 12$

1. Express p as function of n.
   (Note: This means "solve for p".)

2. Express n as a function of p.

Example 7. 1. Given $x - 8y^3 = 0$, express y as function of x.

2. Given $x - 8y^2 = 0$, express y as function of x.

3. Are both things possible? Why or why not?

Example 8. Given $x^2 + y^2 = 1$, express y as function of x, or explain why if you cannot.

<u>**Tables**</u>

Example 9. The following table shows the average high temperature f in San Jose in degrees Fahrenheit in month x.

Month	x	Jan 1	Feb 2	Mar 3	Apr 4	May 5	June 6	July 7	Aug 8	Sept 9	Oct 10	Nov 11	Dec 12
Avg. temp.	$f(x)$	60	64	67	70	75	81	84	84	82	76	67	61

1. Solve for $f(x) = 60$. What does this represent?

2. Find $f(7)$. What does this represent?

3. Solve for $f(x) = 84$. What does this represent?

Example 10. Answer the following given the table:

x	-1	0	1	3	4	6
$f(x)$	6	5	6	1	2	0

1. Find $f(1)$
2. Find $f(6)$
3. Find $f(-2)$
4. Solve $f(x) = 1$
5. Solve $f(x) = 0$
6. Solve $f(x) = 6$

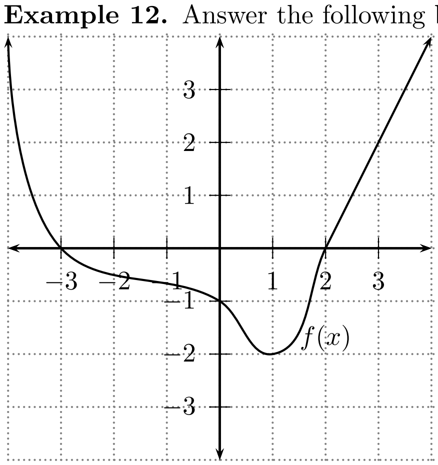
Example 11. Answer the following given the table for $y = f(x)$:

x	0	1	2	3	4
y	-4	-2	0	-2	4

1. Find $f(0)$
2. Find $f(3)$
3. Find $f(-2)$
4. Solve $f(x) = 1$
5. Solve $f(x) = 0$
6. Solve $f(x) = -2$

Graphs

Example 12. Answer the following based on the graph of $f(x)$.



1. Find $f(-3)$
2. Find $f(0)$
3. Find $f(2)$
4. Solve $f(x) = -2$
5. Solve $f(x) = 0$
6. Solve $f(x) = 4$

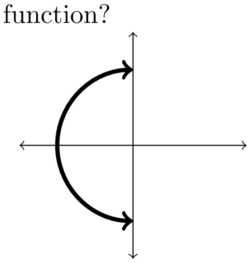
Definitions

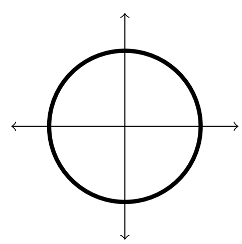
- One-to-one:

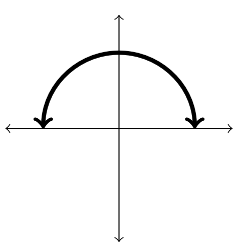
- Vertical Line Test:

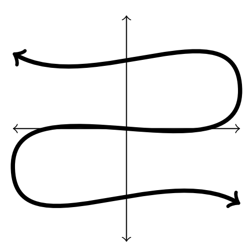
- Horizontal Line Test:

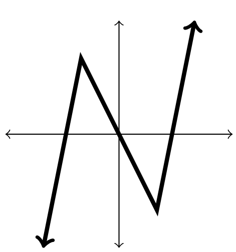
Example 13. For each of the following, determine if the graph represents a function. If it is a function, is it a one-to-one function?

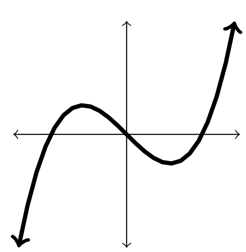












Example 14. For each of the following, determine whether y is a function of x. If it is a function, is it a one-to-one function?

1. $y = x^2$

2. $y = \sqrt{x}$

3. $x^2 + y^2 = 4$

4. $x + y = 4$

5. $x^2 + y = 4$

6. $x + y^2 = 4$

1.3 1.2-1.3: Domain, range and ROC

Domain given an equation

Example 15. *(1.2: Try It #2)* Find the domain of the function $f(x) = 5 - x + x^3$

The **domain** of a **polynomial** is always ____________________________.

Example 16. *(1.2: Try It #3)* Find the domain of the function $f(x) = \dfrac{x+1}{2-x}$

Example 17. *(1.2: Try It #4)* Find the domain of the function $f(x) = \sqrt{5 + 2x}$

The two most important things to keep in mind when finding the domain of an equation are:

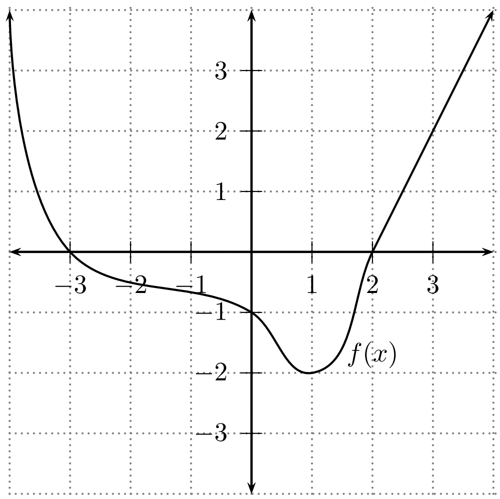
-

-

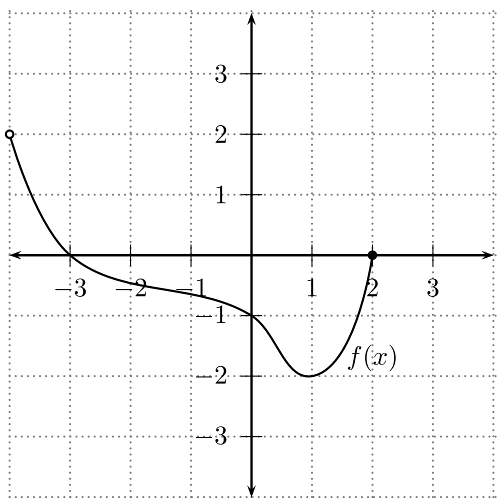
What's the difference between the *mathematical* domain and range and the *practical* domain and range of a function?

Domain and range given a graph

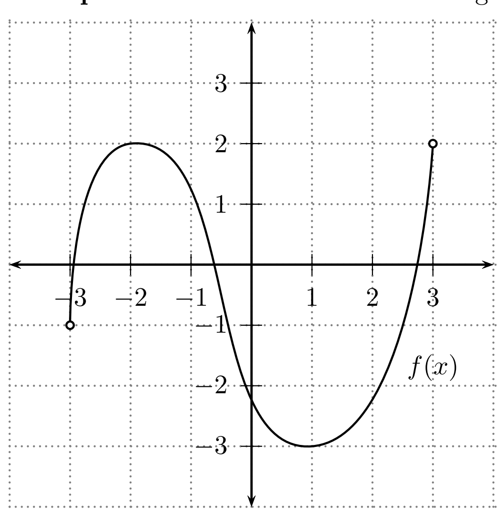
Example 18. Find the domain and range of the function based on its graph.



Example 19. Find the domain and range of the function based on its graph.



Example 20. Find the domain and range of the function based on its graph.



Defintions

- Piecewise function

- $|x| =$

Example 21. Find each of the following for the piecewise function

$$f(x) = \begin{cases} x^2 + 1 & \text{, if } x < 1 \\ x - 1 & \text{, if } x \geq 1 \end{cases}$$

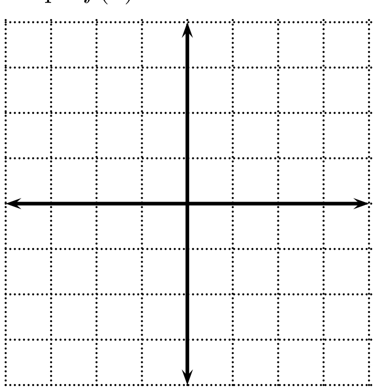
1. $f(-2)$

2. $f(2)$

3. $f(-3)$

4. $f(1)$

5. Graph $f(x)$



Example 22. *(1.2: Try It #8)* Find each of the following for the piecewise function

$$f(x) = \begin{cases} x^3 & \text{, if } x < -1 \\ -2 & \text{, if } -1 < x < 4 \\ \sqrt{x} & \text{, if } x > 4 \end{cases}$$

1. $f(-2)$

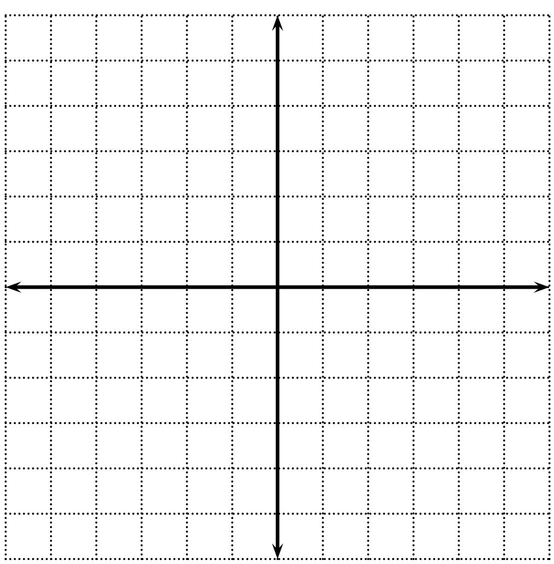
2. $f(2)$

3. $f(-4)$

4. $f(4)$

5. $f(9)$

6. Graph $f(x)$



Definition

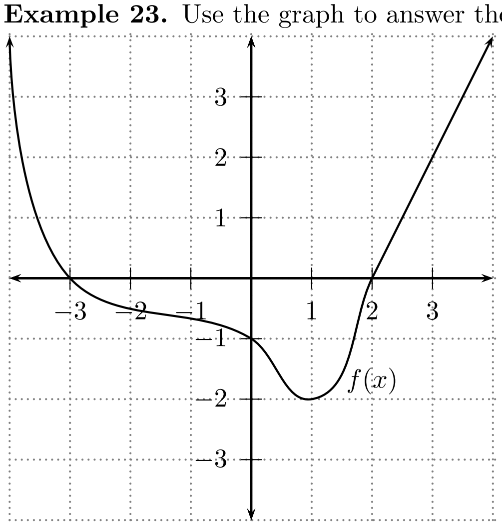
- Average rate of change (ROC)

To compute the average ROC between 2 points we find ______________________________________ connecting those 2 points.

We always need ___________________________ to find an average ROC.

Average ROC and Graphs

Example 23. Use the graph to answer the following questions.



1. Find the average ROC on between $x = 0$ and $x = 3$.

2. Find the average ROC on the interval $[-3, 3]$

3. Find the average ROC on the interval $[-3, 2]$

Average ROC and Tables

Example 24. Answer the following given the table for $y = f(x)$:

x	0	1	2	3	4
y	-4	-2	0	-2	4

Find the average ROC on between $x = 0$ and $x = 4$.

Find the average ROC on between $x = 1$ and $x = 3$.

Average ROC and Equations

Example 25. *(Try It #2)* Find the average ROC of $f(x) = x - 2\sqrt{x}$ on the interval $[1, 9]$

Example 26. *(Try It #3)* Find the average ROC of $f(x) = x^2 + 2x - 8$ on the interval $[5, a]$

Example 27. Find the average ROC of $f(x) = x^2 + 2x - 8$ on the interval $[5, 5 + h]$

Why would we ever need formulas like the ones we came up with in Examples 26 and 27?

1.4 1.4: Composition of functions

Combining Functions with Equations

Example 28. Find each of the following for $f(x) = x^2$ and $g(x) = 1 - x$

1. $(f + g)(x)$

2. $(f + g)(2)$

3. $(f - g)(x)$

4. $(g - f)(x)$

5. $(f - g)(2)$

6. $(g - f)(2)$

7. $(fg)(x)$

8. $(gf)(x)$

9. $\left(\frac{f}{g}\right)(x)$

10. $\left(\frac{g}{f}\right)(x)$

11. Find the domain of $\left(\frac{f}{g}\right)(x)$

12. Find the domain of$\left(\frac{g}{f}\right)(x)$

Composition of Functions with Equations

Notation: We can write the composition on functions "f of g of x" in 2 ways:

Example 29. Find each of the following for $f(x) = \sqrt{2x+5}$ and $g(x) = 4x^2 + 1$

1. $(f \circ g)(x)$

2. $(g \circ f)(x)$

3. $(f \circ g)\left(-\frac{1}{2}\right)$

4. $(g \circ f)\left(-\frac{1}{2}\right)$

5. Find the domain of $f(x),\ g(x),\ (f \circ g)(x)$ and $(g \circ f)(x)$

Applications of Composition of Functions

Example 30. *(Example 4)* Suppose $f(x)$ is the number of miles you can drive in x hours and $g(y)$ gives the gallons of gas used in driving y miles.

- Does $f(g(y))$ make sense? What does it represent?

- Does $g(f(x))$ make sense? What does it represent?

Example 31. *(Try It #2)* The gravitational force on a planet r miles from the sun is given by $G(r)$. The acceleration of a planet subjected to any force is given by $a(F)$.

- Does $G(a(F))$ make sense? What does it represent?

- Does $a(G(r))$ make sense? What does it represent?

Composition of Functions Using Tables

Example 32. Complete the table.

x	$f(x)$	$g(x)$	$f(g(x))$	$g(f(x))$	$f(f(x))$
0	2	4			
1	5	4			
2	3	0			
3	3	-1			
4	0	8			
5	9	1			

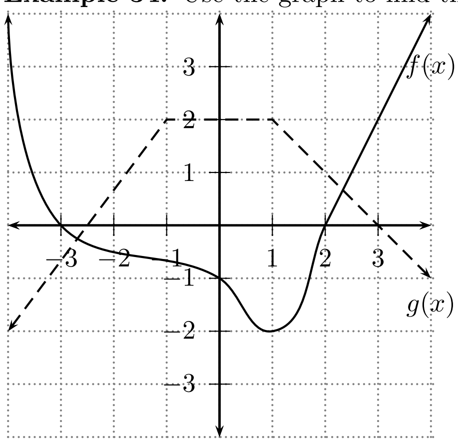
Example 33. Use the table to find the following.

x	-1	0	1	3	4	6
$f(x)$	6	4	6	1	2	0
$g(x)$	-1	1	-1	0	2	7

1. $f(g(-1))$
2. $g(f(-1))$
3. $f(g(6))$
4. $g(f(3))$

Composition of Functions Using Graphs

Example 34. Use the graph to find the following



1. $f(g(1))$
2. $f(g(-1))$
3. $f(g(2))$
4. $g(f(0))$
5. $g(f(3))$

Example 35. Answer the following for $f(x) = \dfrac{5}{x-1}$ and $g(x) = \dfrac{4}{x}$.

1. Find $f(g(x))$

2. Find the domain of $f(g(x))$

3. Find $(g(f(x))$

4. Find the domain of $(g(f(x))$

5. Find $(g(f(6))$

Example 36. Answer the following for $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{3-x}$.

1. Find $f(g(x))$

2. Find the domain of $f(g(x))$

3. Find $(g(f(x))$

4. Find the domain of $(g(f(x))$

5. Find $(f(g(-1))$

Example 37. Answer the following for $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x+4}$.

1. Find $f(g(x))$

2. Find the domain of $f(g(x))$

3. Find $(f(g(5))$

Chapter 2

Transformations, absolute value, and inverses

2.1 1.5A: Transformations of functions

For each of the transformation types below, write notes on what you learned about it from the group activity.

- $g(x) = f(x) + k$
  - What is this graph translation called?
  - How do we use the graph of $f(x)$ to determine the graph of $g(x)$?
  - What happens if k is positive? Negative?
- $g(x) = f(x + k)$
  - What is this graph translation called?
  - How do we use the graph of $f(x)$ to determine the graph of $g(x)$?
  - What happens if k is positive? Negative?
  - Note: Your textbook writes this translation in general as $g(x) = f(x - h)$.
- $g(x) = -f(x)$
  - What is this graph translation called?
  - How do we use the graph of $f(x)$ to determine the graph of $g(x)$?

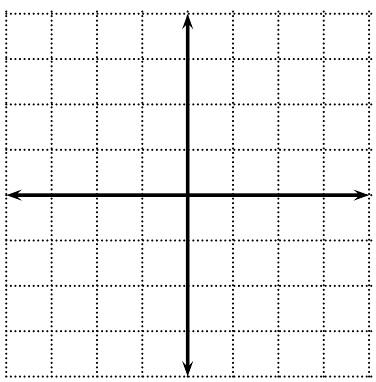
- $g(x) = f(-x)$
  - What is this graph translation called?
  - How do we use the graph of $f(x)$ to determine the graph of $g(x)$?
- $g(x) = a \cdot f(x)$
  - What is this graph translation called?
  - How do we use the graph of $f(x)$ to determine the graph of $g(x)$?
  - What happens if $a > 1$? If $0 < a < 1$?
  - What happens if a is negative?
- $g(x) = f(b \cdot x)$
  - What is this graph translation called?
  - How do we use the graph of $f(x)$ to determine the graph of $g(x)$?
  - What happens if $b > 1$? If $0 < b < 1$?
  - What happens if b is negative?

The **rigid** graph transformations are:

The **nonrigid** graph transformations are:

Given any transformation $a \cdot f(bx+h)+k$, what is the **order** that you should perform the transformations in?

Example 38. Sketch the graph of $g(x) = (2x+3)^2 + 2$ by first sketching its parent function. You may want to list your transformations and/or sketch multiple transformation graphs.



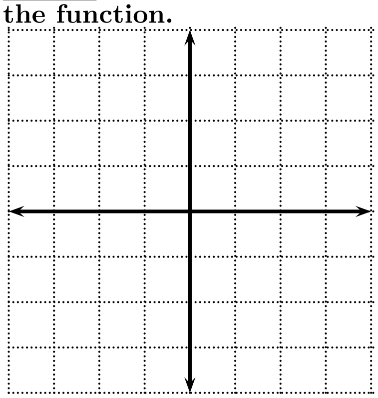
2.2 1.5B: More Practice with Graph Transformations

Example 39. Graph $f(x) = \sqrt{-x-1}$

Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

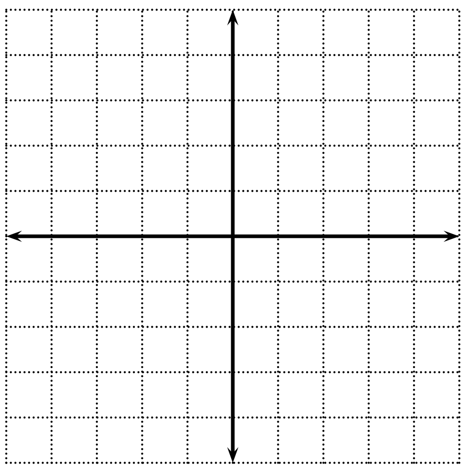
Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**



GROUP WORK: PRACTICE ON GRAPH TRANSLATIONS

Example 40. Graph $g(x) = 2 - 8x^4$
Step 1: Identify the parent function

Step 2: Identify the transformations, in order.
Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**
(Hint: Choose an appropriate scale for the axes.)

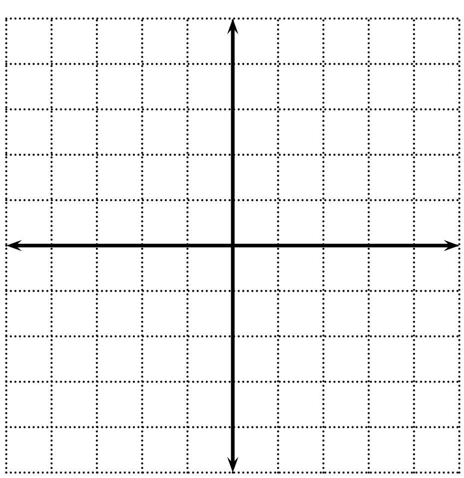


Example 41. Graph $g(x) = \sqrt{3x} + 1$

Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**



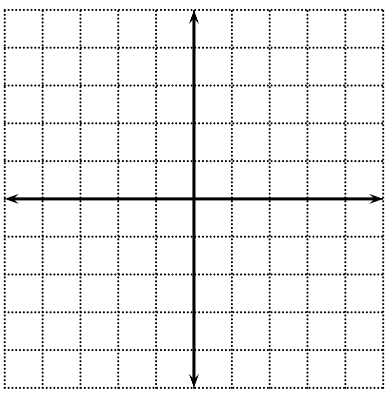
GROUP WORK: PRACTICE ON GRAPH TRANSLATIONS

Example 42. Graph $g(x) = \frac{1}{2}|2 - x| - 3$

Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**

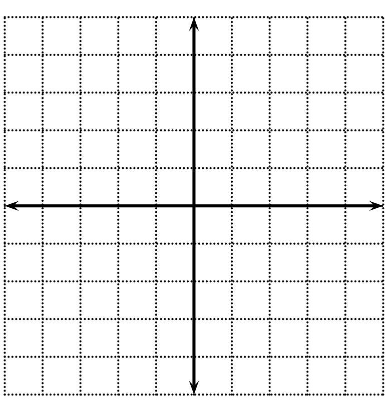


Example 43. Graph $g(x) = 2(x - 7)^2$

Step 1: Identify the parent function

Step 2: Identify the transformations, in order.

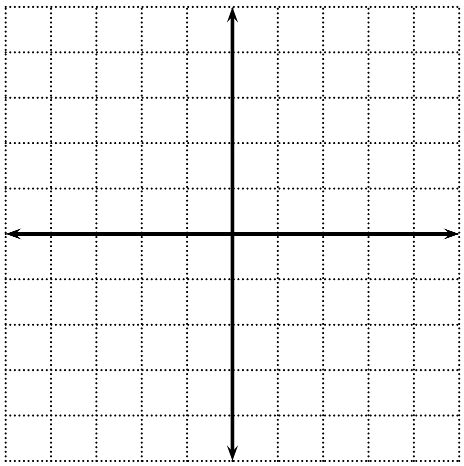
Step 3: Sketch the graph, including any intermediate graphs you find helpful. **Clearly identify the final graph of the function.**



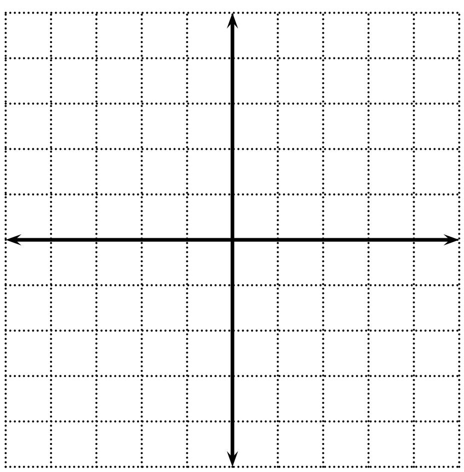
GROUP WORK: PRACTICE ON GRAPH TRANSLATIONS

For each of the following functions, identify the parent function, identify the necessary transformations, in order, and sketch a graph of the function. You may include any intermediate graphs you find helpful, but clearly identify the final graph of the function.

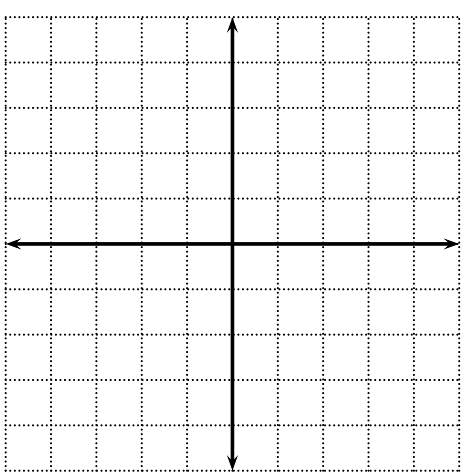
1. $g(x) = -\frac{1}{4}(x+2)^2 - 2$



2. $g(x) = 1 - \sqrt{\frac{1}{4}x}$



3. $g(x) = 6 - |-x+3|$



2.3 1.6: Solving Absolute Value Equations and Inequalities

Example 44. Describe the inequality $|x-3| \leq 2$ verbally, then graph it on the number line and write the solution in interval notation.

Example 45. Describe the inequality $|x-1| > 5$ verbally, then graph it on the number line and solve it algebraically. Write the solution in interval notation.

Example 46. a) Solve $|2x-1| = 3$

b) Solve $|2x-1| < 3$

Example 47. Solve $|5x+2|-4=9$

Example 48. *(Try It #4)* Solve $|2x-1|-3=0$

Example 49. Solve $-|x+2|+3=0$

Example 50. Solve $|5x+2|-4=9$

Example 51. *(Try It #2)* Students who score within 20 points of 80 will pass a test. Write this as a distance from 80 using absolute value notation.

Example 52. *(Example 7)* Solve $-\dfrac{1}{2}|4x-5|+3<2$

Example 53. Solve $2+|3x-5|\leq 1$

Example 54. *(Try It #7)*

a) Solve $-2|k-4|\leq -6$

b) Solve $-2|k-4|>1$

c) Solve $-2|k-4|<1$

d) Solve $-2|k-4|<0$

2.4 1.7: Inverse functions

<u>Definition</u>:

<u>Inverses from formulas</u>:

Example 55. *(Try It #2)* True or false: If $f(x) = x^3 - 4$, then $f^{-1}(x) = \sqrt[3]{x-4}$

Example 56. *(Try It #3)* True or false: If $f(x) = (x-1)^3$, then $f^{-1}(x) = \sqrt[3]{x} + 1$

Example 57. *(Example 7)* The formula to convert a Fahrenheit temperature, F, to a Celsius temperature, C, is given by

$$C = \frac{5}{9}(F - 32)$$

Find a formula to convert a Celsius temperature to a Fahrenheit temperature by finding the inverse of the function.

Example 58. Find the inverse of each function. Then find the domain and range of both the function and its inverse.

1. *(Try It #7)* $p(x) = \frac{1}{3}(x - 5)$

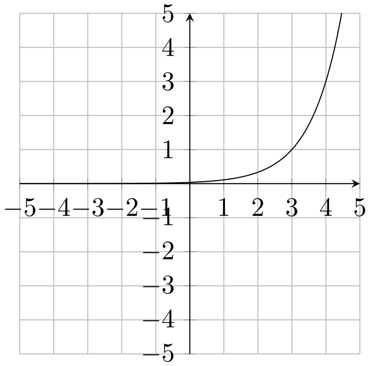
2. $f(x) = 1 - \sqrt{2x - 3}$

3. $g(x) = \frac{5x - 3}{2x + 5}$

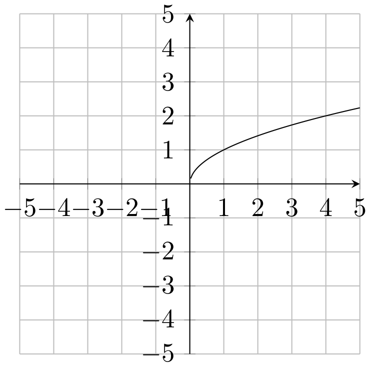
4. $h(x) = \frac{1}{x + 3} + 2$

Inverse from graphs:

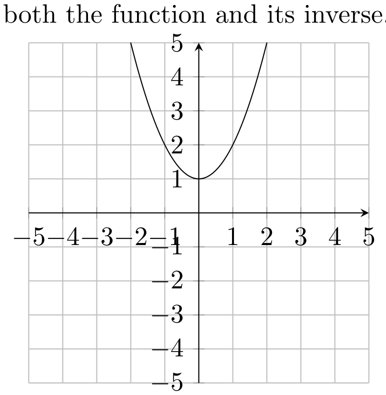
Example 59. Given the graph of $f(x) = 3^{x-3}$ below, sketch the graph of $f^{-1}(x)$. Then find the domain and range of both the function and its inverse.



Example 60. Given the graph of $g(x) = \sqrt{x}$ below, sketch the graph of $g^{-1}(x)$. Then find the domain and range of both the function and its inverse.



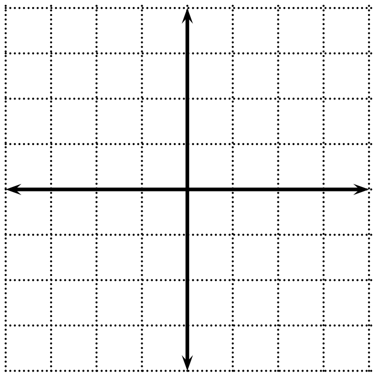
Example 61. Given the graph of $h(x) = x^2 + 1$ below, sketch the graph of $h^{-1}(x)$. Then find the domain and range of both the function and its inverse.



GROUP WORK: INVERSES OF COMMON PARENT FUNCTIONS

For each parent function $f(x)$, find the inverse function $f^{-1}(x)$ and sketch the graphs of both functions. Then find the domain and range for both $f(x)$ and $f^{-1}(x)$. For some parent functions, you may need to restrict the domain of $f(x)$ to make it invertible. In such cases, write the restricted domain and only sketch the graph on this domain.

1. $f(x) = x$

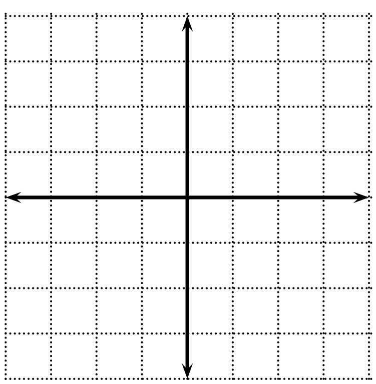


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

2. $f(x) = |x|$

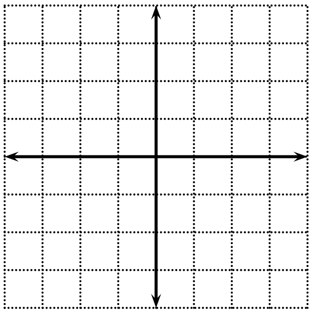


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

3. $f(x) = x^2$

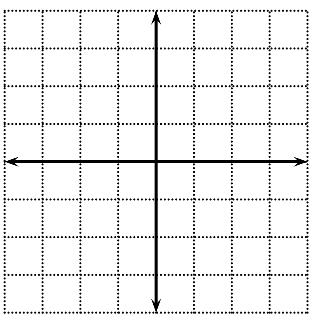


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

4. $f(x) = x^3$

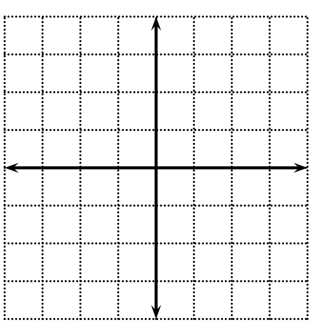


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

5. $f(x) = \dfrac{1}{x}$

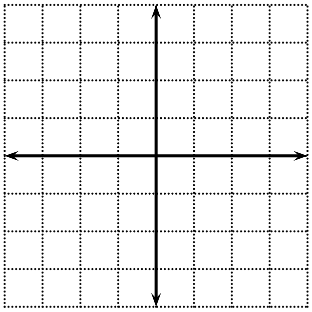


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

6. $f(x) = \sqrt{x}$

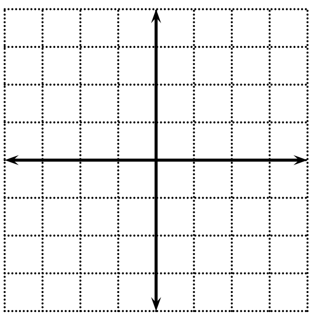


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

7. $f(x) = \sqrt[3]{x}$

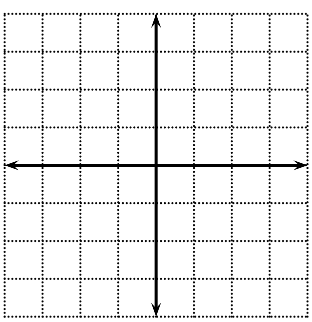


$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

Bonus: $f(x) = \dfrac{1}{x^2}$



$f^{-1}(x) =$ ______________________________

Domain of $f(x)$: __________________ Domain of $f^{-1}(x)$: __________________

Range of $f(x)$: __________________ Range of $f^{-1}(x)$: __________________

Chapter 3

Linear Functions

3.1 2.1: Linear functions

What does it mean for a function to be **linear** as a...

- Graph?

- Table?

x	-1	0	1	2	3
$f(x)$					

- Equation?

- Verbal expression?

Example 62. *(Example 1)* The pressure, P, in pounds per square inch (psi) on a diver depends on her depth, d, below the surface of the water, in feet, modeled by the equation $P(d) = 0.434d + 14.696$. Restate this function in words.

Definitions:

- Slope

- Increasing

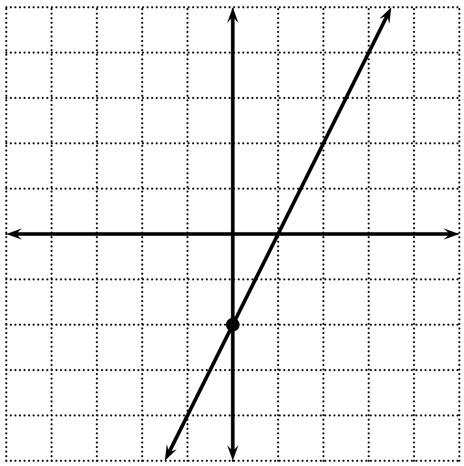
- Decreasing

- Constant

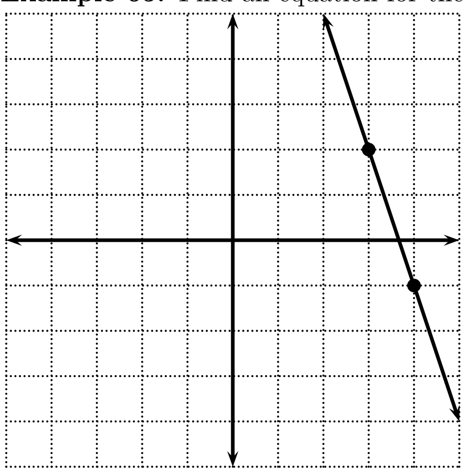
The **units of the slope** are always:

Example 63. *(Try It #2)* The population of a small town increased from 1,442 to 2,868 between 2009 and 2012. Find the rate of change of the population per year if we assume the change was constant during this time period.

Example 64. Find an equation for the linear function in the graph.



Example 65. Find an equation for the linear function in the graph.



Example 66. Write an equation for the line that passes through the points $(-1,3)$ and $(1,0)$.

Example 67. Write an equation for the line that passes through the points $(0,7)$ and $(4,4)$.

Example 68. If $f(x)$ is a linear function with $f(2) = -11$ and $f(4) = -25$, find an equation for the function.

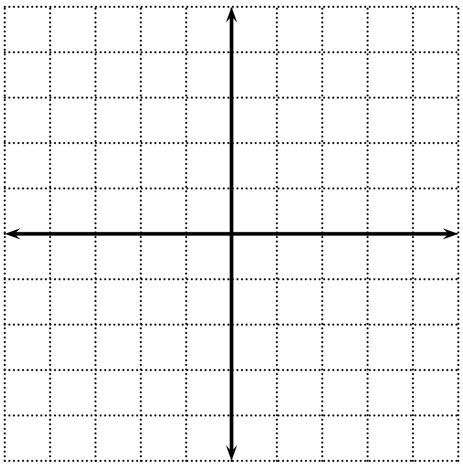
Example 69. *(Example 11)* Working as an insurance salesperson, Ilya earns a base salary plus a commission on each new policy. So Ilya's weekly income depends, I, on the number of new policies, n, that he sells during the week. Last week he sold 3 new policies and earned \$760 for the week. The week before, he sold 5 new policies and earned \$920. Find an equation for $I(n)$, and interpret the meaning of the equation.

Example 70. *(2.3 Try It #1)* A company sells donuts. They incur a fixed cost of \$25,000 or rent, insurance and other expenses. It costs \$0.25 to produce each donut. Write a linear model to represent the cost C of the company as a function of the number of donuts produced, x. Then find and interpret the vertical intercept of the function.

Example 71. Back to **Example 63**, find a linear equation $P(x)$ to model the population of the town, where x is the the number of years after the year 2009. The use your model to predict what the population of the town was in 2015.

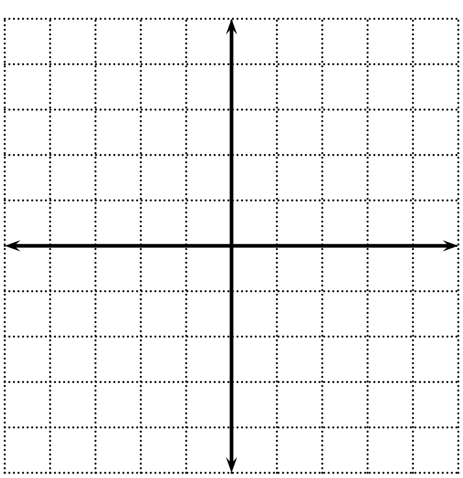
3.2 2.2: Linear graphs

Example 72. Sketch a graph of the linear function $f(x) = -\frac{3}{4}x + 6$

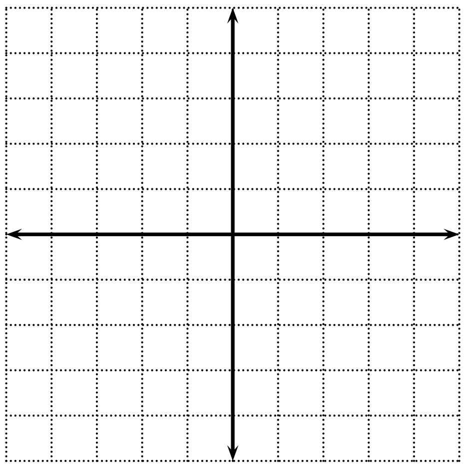


Do all linear functions have a vertical intercept?

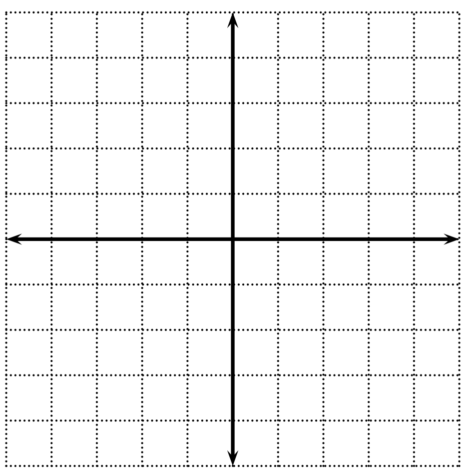
Example 73. Sketch a graph of the linear function $f(x) = \frac{2}{3}x - 2$



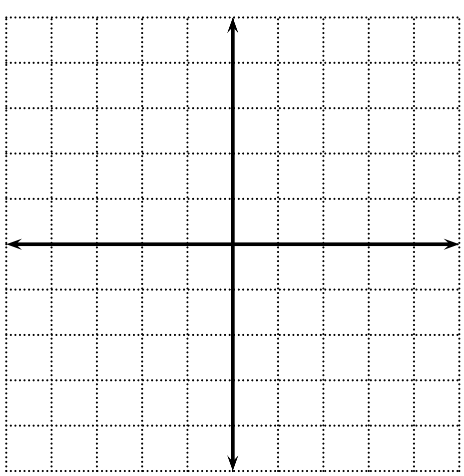
Example 74. Sketch a graph of the linear function $g(x) = \frac{1}{2}x - 3$



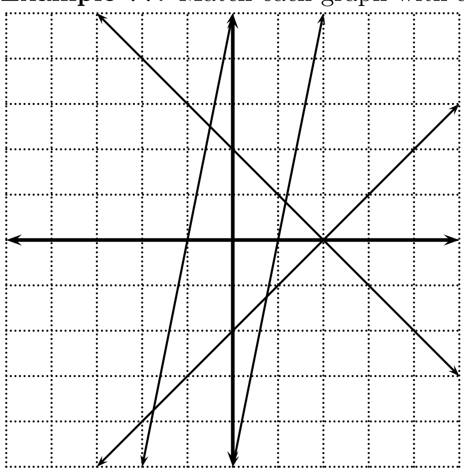
Example 75. *(Try It #3)* Sketch a graph of the linear function $g(x) = 4 + 2x$



Example 76. Sketch a graph of the linear function $g(x) = -3x$



Example 77. Match each graph with an equation

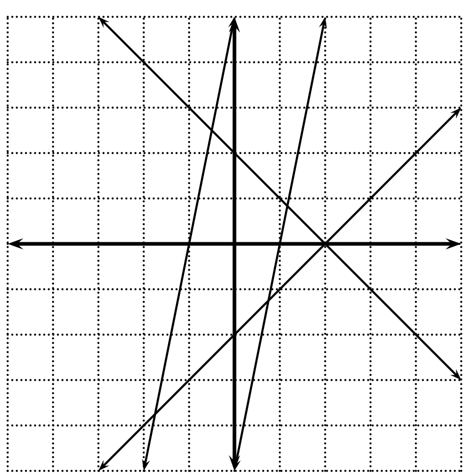


$f(x) = 5x + 5$ $g(x) = 2 - x$ $h(x) = 5x - 5$ $p(x) = x - 2$

Example 78. *(Try It #4)*

1. Find the vertical and horizontal intercepts of the linear function $f(x) = \dfrac{1}{4}x - 4$.

2. Use the vertical and horizontal intercepts to graph $f(x)$.

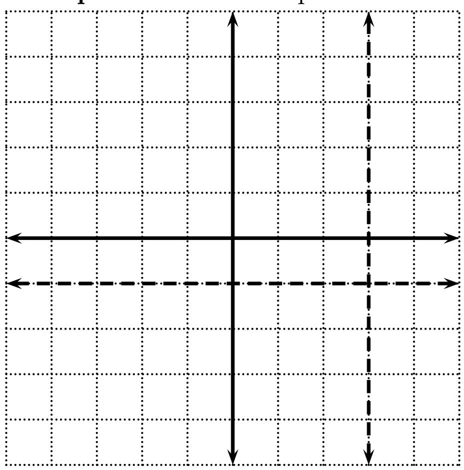


3. Use m and b in the equation for $f(x)$ to check your work in Part (b).

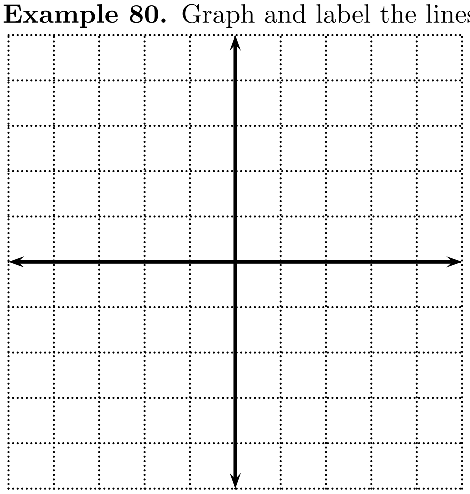
Do all linear functions have a horizontal intercept?

Horizontal and Vertical Lines

Example 79. Find an equation for each of the dashed lines. Do the lines represent functions?



Example 80. Graph and label the lines $y = 3$ and $x = -2$.



Parallel and perpendicular lines:

- Parallel lines:

- Perpendicular lines:

Example 81. *(Example 8)* Given the linear functions below, identify any pairs of functions whose graphs are parallel or perpendicular.

$f(x) = 2x + 3$ $\quad g(x) = \frac{1}{2}x - 4$ $\quad h(x) = -2x + 2$ $\quad p(x) = 2x - 6$

Chapter 4

Complex numbers, parabolas and polynomials

4.1 3.1: Complex numbers

Definition: $i =$

Example 82. Simplify the following radicals.

1. $\sqrt{-16}$

2. $\sqrt{-81}$

3. *(Try It #1):* $\sqrt{-24}$

4. $\sqrt{-4}$

5. $\sqrt{-5}$

6. $\sqrt{-18}$

Definitions:

- **Imaginary number:**

- **Complex number:**

Multiplying Complex Numbers:

Example 83. Multiply each of the following.

1. *(Try It #5)* $(3-4i)(2+3i)$

2. $(2+5i)(4-i)$

3. $(2+i)(3+i)$

Definition: Complex conjugate:

Example 84. Find the conjugate of each of the following complex numbers.

1. $2+\sqrt{5}i$
2. $3-i$
3. $-2i$

Example 85. Multiply each of the complex conjugate pairs in Example 84.

1.

2.

3.

Whenever we multiply a pair of complex conjugates the result is ____.

Example 86. *(Try It #6)* If $f(x) = 2x^2 - 3x$, find $f(8 - i)$.

Example 87. If $g(x) = x^2 - 5x + 2$, find $g(3 + 2i)$.

Example 88. If $h(x) = x^2 + 1$, find $h(i)$.

4.2 3.2A: Parabolas and quadratic equations

Definitions:

What is a **quadratic function**?

- Equation:
  - Parent function:
  - Vertex form:
  - General form:
- Graph: **Parabolas**
- Verbal applications:

Vocabulary and properties of parabolas

Goal: Given a quadratic equation in general form $f(x) = ax^2 + bx + c$, can we find the vertex? Horizontal and vertical intercepts? Graph the parabola?

Quadratic formula:

So given $f(x) = ax^2 + bx + c$, we can use ____________________________ to find any **horizontal intercepts** and

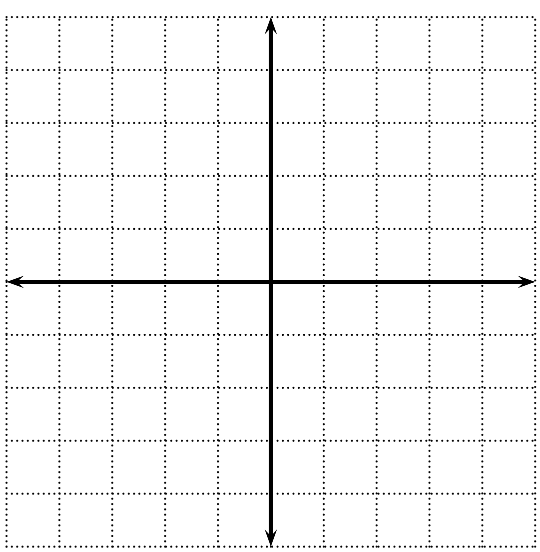
____________________________ to find the **vertical intercept**.

Axis of symmetry:

Vertex:

Example 89. Find the following for $f(x) = 2x^2 - 6x + 7$.

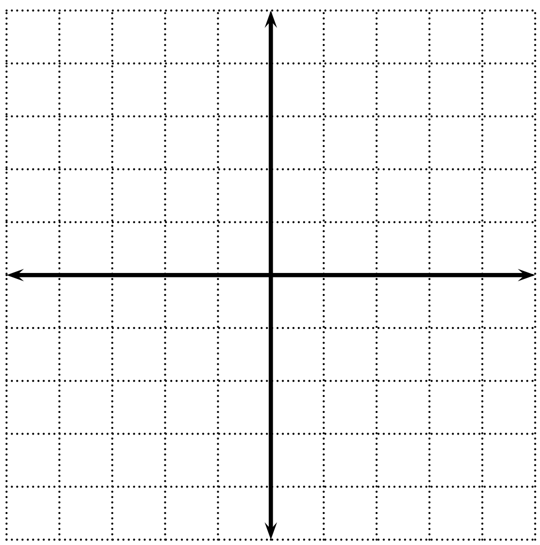
- Axis of symmetry:
- Vertex:
- Opens up or down?
- How many real roots (horizontal intercepts) are there?
- Vertical intercept:
- Sketch the graph. (Choose an appropriate scale for the axes.)



- Use the *discriminant* to check the number of horizontal intercepts.

Example 90. Find the following for $g(x) = 13 + x^2 - 6x$.

- Axis of symmetry:
- Vertex:
- Opens up or down?
- How many real roots (horizontal intercepts) are there?
- Vertical intercept:
- Sketch the graph. (Choose an appropriate scale for the axes.)

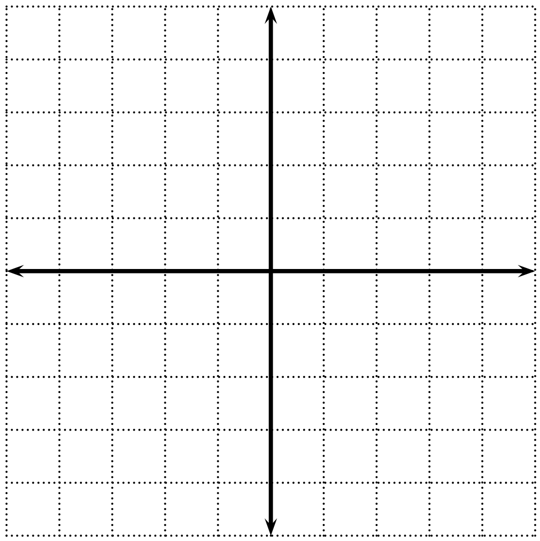


- Domain:
- Range:

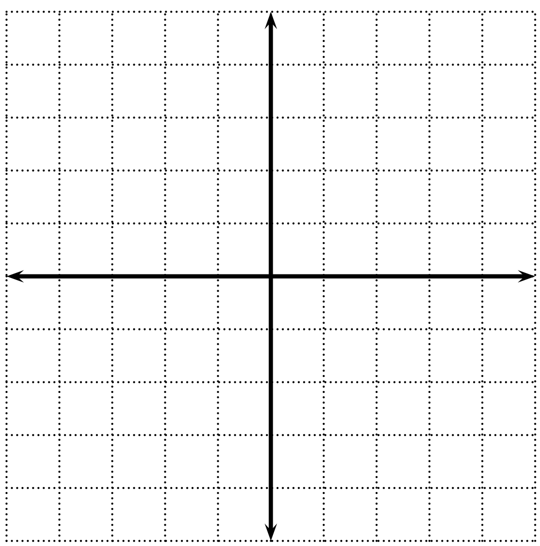
Graphing from vertex form

Example 91.

- Graph the quadratic function $f(x) = -3(x-2)^2 + 5$ by translating the parent function $y = x^2$.

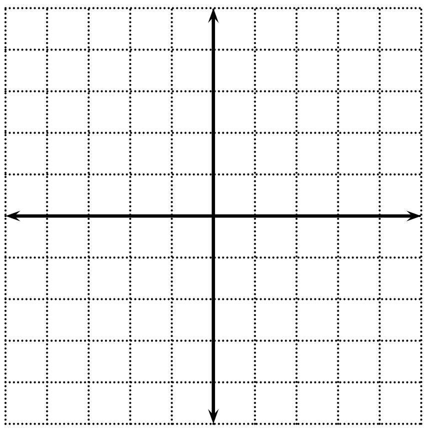


- Graph the same quadratic function $f(x) = -3(x-2)^2 + 5$ using the properties of the **vertex form** $a(x-h)^2 + k$

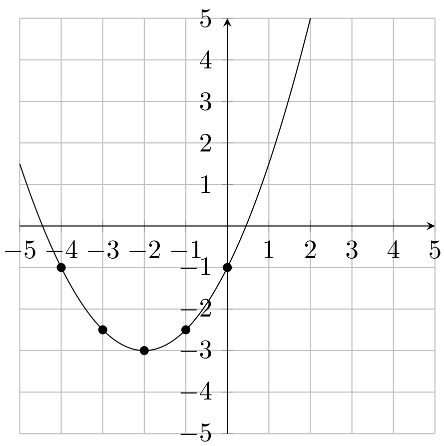


- Find the domain and range of $f(x)$.

Example 92. *(Try It #3)* Graph the quadratic function $f(x) = 2\left(x - \frac{4}{7}\right)^2 + \frac{8}{11}$. (Choose an appropriate scale for the axes.)



Example 93. Find a quadratic equation in vertex form for the parabola graphed below.



Maxima and minima:

Converting between General form and Vertex form

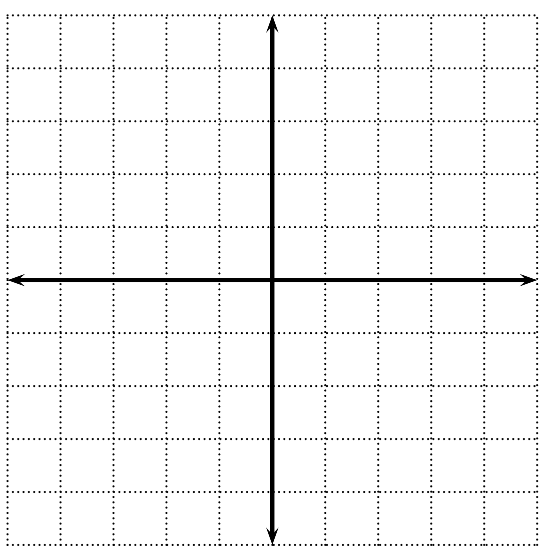
Example 94. Write $f(x) = -2(x-1)^2 + 3$ in general form.

Example 95. Write $f(x) = -2x^2 + 4x + 1$ in vertex form.

GROUP WORK: PRACTICE GRAPHING PARABOLAS

Example 96. Find the following for $f(x) = x^2 - 4x + 3$.

- Axis of symmetry:
- Vertex:
- Opens up or down?
- How many real roots (horizontal intercepts) are there?
- Use the *discriminant* to check the number of horizontal intercepts and find them if they exist.
- Vertical intercept:
- Sketch the graph. (Choose an appropriate scale for the axes.)

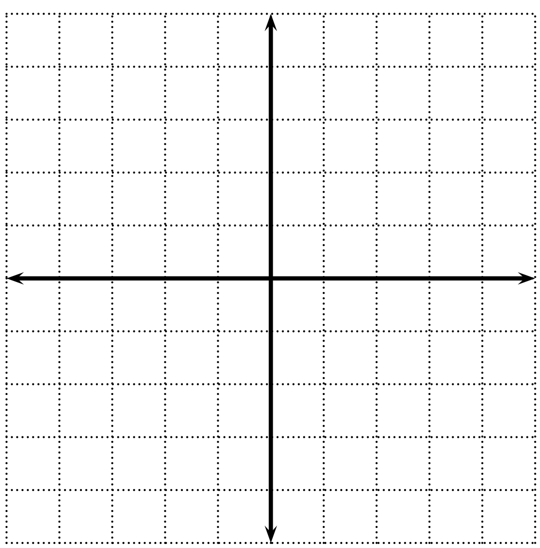


- Domain:
- Range:

GROUP WORK: PRACTICE GRAPHING PARABOLAS

Example 97. Find the following for $f(x) = 3x^2 - 6x + 4$.

- Axis of symmetry:
- Vertex:
- Opens up or down?
- How many real roots (horizontal intercepts) are there?
- Use the *discriminant* to check the number of horizontal intercepts and find them if they exist.
- Vertical intercept:
- Sketch the graph. (Choose an appropriate scale for the axes.)

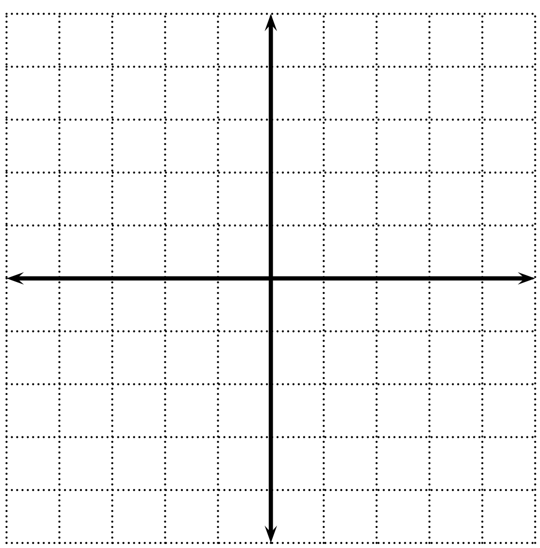


- Domain:
- Range:

GROUP WORK: PRACTICE GRAPHING PARABOLAS

Example 98. Find the following for $f(x) = -x^2 - 4x + 1$.

- Axis of symmetry:
- Vertex:
- Opens up or down?
- How many real roots (horizontal intercepts) are there?
- Use the *discriminant* to check the number of horizontal intercepts and find them if they exist.
- Vertical intercept:
- Sketch the graph. (Choose an appropriate scale for the axes.)



- Domain:
- Range:

4.3 3.2B: Factoring and Roots of Parabolas

Greatest Common Factor

Example 99. For each expression, factor out the GCF.

1. $5x^2 - 15x$

2. $-3 + 6x - 12x^2$

3. $4x^2 + 16x$

Difference of Squares

Example 100. Factor $x^2 - 81$

Example 101. Factor $100 - 4y^2$

Perfect Squares $(a \pm b)^2$

Example 102. Factor $x^2 - 10x + 25$

Example 103. Factor $9x^2 + 30x + 25$

Example 104. Factor $4y^2 - 12y + 9$

Factoring Trinomials (Big X)

Example 105. Factor $12x^2 + 7x + 1$

Example 106. Factor $3b^2 - 10b - 8$

Example 107. Factor $x^2 + x - 6$

Example 108. Solve $-x^2 - 2x + 35 = 0$

GROUP WORK: PRACTICE ON FACTORING

1. A student submitted the following **incorrect** work to simplify an expression. Find where the student made their error(s), then redo the problem correctly.

$$\begin{aligned} 6x^2 - 5x - 4 &= 6x^2 + 3x - 8x - 4 \\ &= 3x(2x+1) - 4(2x-1) \\ &= (2x+1)(2x-1)(3x-4) \end{aligned}$$

Factor each trinomial with leading coefficient $a \neq 1$.

2. $2x^2 + 9x + 7$

3. $5y^2 - 16y + 3$

Solve by factoring and using the zero product principle.

4. $8x^2 - 18x + 9 = 0$

5. $6x^2 - 5x - 6 = 0$

GROUP WORK: PRACTICE ON FACTORING

Completely factor each trinomial. Make sure you factor out the GCF first if there is one.

6. $5p^2 - 5p - 60$

7. $2t^2 - 12t - 32$

Solve by factoring. Make sure you factor out the GCF first if there is one.

8. $3b^2 - 2b - 6 = 0$

9. $60 - 40y + 5y^2 = 0$

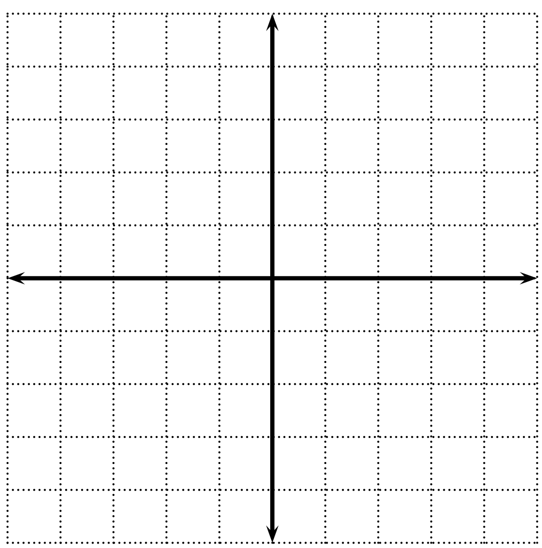
Example 109. *(Example 7)* Answer the following for $f(x) = 3x^2 + 5x - 2$.

1. Find the horizontal intercepts by factoring.

2. Find the vertical intercept.

3. Find the vertex.

4. Sketch the graph. (Choose an appropriate scale for the axes.)



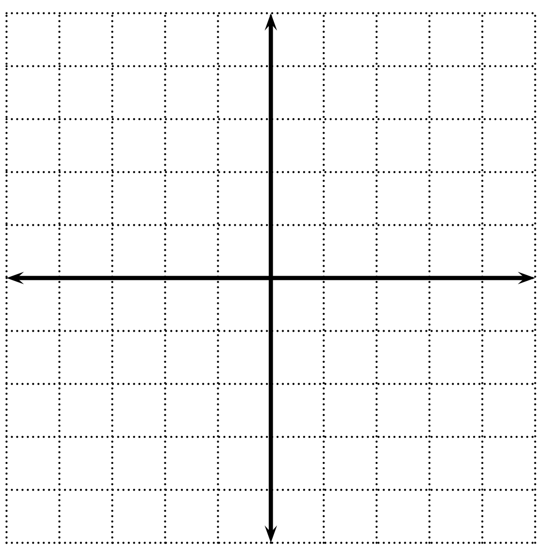
Example 110. *(Example 8)* Answer the following for $f(x) = 2x^2 + 4x - 4$.

1. Find the horizontal intercepts.

2. Find the vertical intercept.

3. Find the vertex.

4. Sketch the graph. (Choose an appropriate scale for the axes.)



Applications of quadratic functions

To find a maximum or minimum value:

To find when something hits the ground or lands:

To find the value something starts at:

Example 111. *(Example 5)* A backyard farmer wants to enclose a rectangular space for a new garden within her fenced backyard. She has purchased 80 feet of wire fencing to enclose the 3 sides, and she will use use a section of the backyard fence as the fourth side.

(a) Find a formula for the area enclosed by the fences. Draw a diagram and label any variables you use.

(b) What dimensions should she make her garden to **maximize** the enclosed area.

(c) **In words**, write a sentence explain how you know that the dimensions you found in Part (c) **maximize** the area.

Example 112. *(Example 10)* If a ball is thrown upward from the top of a 40 foot high building at a speed of 80 feet per second, the ball's height, H, above the ground in feet can be modeled by the function $H(t) = -16t^2 + 80t + 40$, where t is the time, in seconds, since the ball was thrown.

1. When does the ball reach its **maximum** height?

2. What is the maximum height of the ball?

3. When does the ball hit the ground?

Example 113. *(Try It #5)* If a rock is thrown upward from the top of a 112 foot high cliff overlooking the ocean at a speed of 96 feet per second, the rock's height, H, above the ocean in feet can be modeled by the function $H(t) = -16t^2 + 96t + 12$, where t is the time, in seconds, since the rock was thrown.

1. When does the rock reach its **maximum** height?

2. What is the maximum height of the rock?

3. When does the rock hit the ocean?

GROUP WORK: PRACTICE ON MAX/MIN AND HORIZONTAL INTERCEPTS

Find the vertex of each of the following quadratic equations and decide if it is a **maximum** or a **minimum**. Then find the **horizontal intercepts**, if there are any.

1. $f(x) = x^2 + 4x - 32$

2. $f(x) = 9x^2 - 18x + 3$

3. $f(x) = 2x^2 + 5x - 8$

4. $f(x) = -x^2 + 2x + 7$

HOMEWORK: FACTORING

Completely factor each expression.

1. $x^2 - 7x - 18$
2. $x^2 - 5x - 14$
3. $7x^2 - 31x - 20$
4. $2x^2 + 17x + 21$
5. $9x^2 - 5x - 10$

Solve by factoring.

6. $7x^2 + 9x = 0$
7. $7x^2 - 45x - 28 = 0$
8. $5x^2 - x - 18 = 0$
9. $9x^2 + 7x - 56 = 0$

4.4 3.3-3.4: Polynomials and Their Graphs

Power Functions

Definitions

- **Power function:**
- **Coefficient:**
- **Degree:**
- Examples:

Which of our common parent functions are power functions?

Example 114. *(3.3: Try It #1)* Which of the following are power functions?

1. $f(x) = 2x^2 \cdot 4x^3$
2. $g(x) = -x^5 + 5x^3 - 4x$
3. $h(x) = \dfrac{2x^5 - 1}{3x^2 + 4}$

End Behavior of Power Functions		
	Even power	**Odd power**
Positive constant $k > 0$		
Negative constant $k < 0$		

Domain and Range of Power Functions

- Domain:

- Range:

Example 115. *(3.3: Try It #2)* Describe in words and symbols the end behavior of $f(x) = -5x^4$

Polynomials
Definitions:

- **Polynomial**

- **Term**

- General form

- Leading term

- Degree of a polynomial

- Leading coefficient

- Constant term

Example 116. *(3.3: Example 4)* Which of the following are polynomial functions?

1. $f(x) = 2x^3 \cdot 3x + 4$

2. $g(x) = x(x^2 - 4)$

3. $h(x) = 5\sqrt{x} + 2$

Example 117. *(3.3: Try It #3)* Identify the degree, leading term and leading coefficient of the polynomial

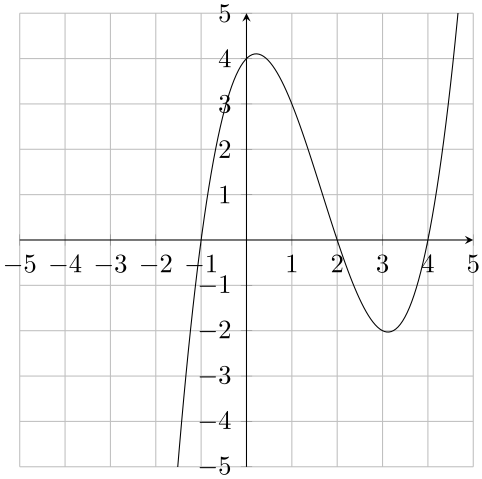
$$f(x) = 4x^2 - x^6 + 6x - 6$$

Graphs of polynomials

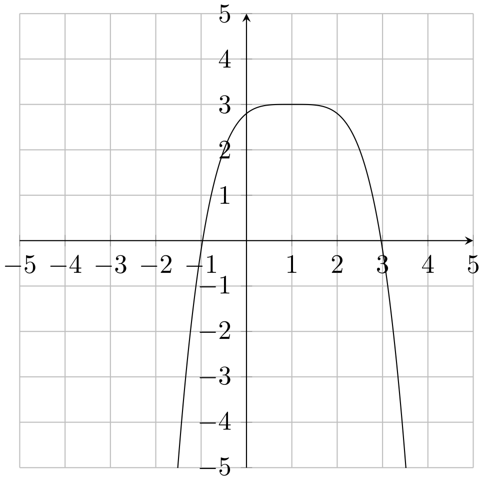
The *degree*, *leading coefficient* and *constant* term tell us a lot about the *graph* of a polynomial

- Constant term:
- End behavior:
- Roots
- Turning points

Example 118. Identify the real roots, turning points, constant term and end behavior on the graph of the polynomial below.



Example 119. *(3.3: Try It #4)* Describe the end behavior and roots of the polynomial graphed below. Then determine a possible degree and possible equation for the function.



- End behavior:
- Roots:
- Possible degree
- Possible equation:
  - Using graph transformations:
  - Using roots:

Example 120. *(3.3: Try It #5)* Answer the following for the polynomial function

$$f(x) = 0.2(x-2)(x+1)(x-5)$$

1. The book asks us to find the leading term by first writing the polynomial in general form. But can we find the leading term **without** multiplying everything out?

2. Degree:

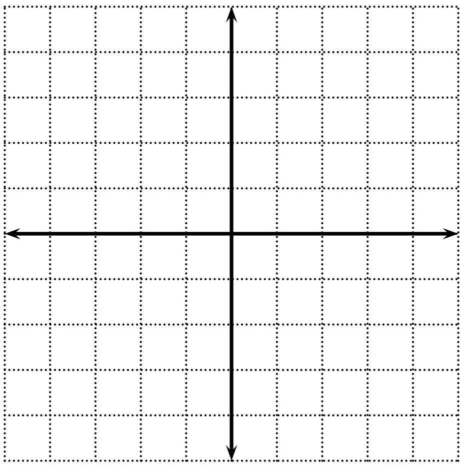
3. End behavior:

4. What benefits, if any, are there to writing a polynomial function in **factored form** like this one is?

5. Horizontal intercepts:

6. Vertical intercept:

Is the information in Example 120 enough to sketch a graph of the polynomial function $f(x)$?



Important notes on graphing polynomial functions:

-

-

-

Example 121. *(3.3: Try It #6)* Answer the following for the polynomial function

$$g(x) = 2x^3 - 6x^2 - 20x$$

1. **Factor** the polynomial:

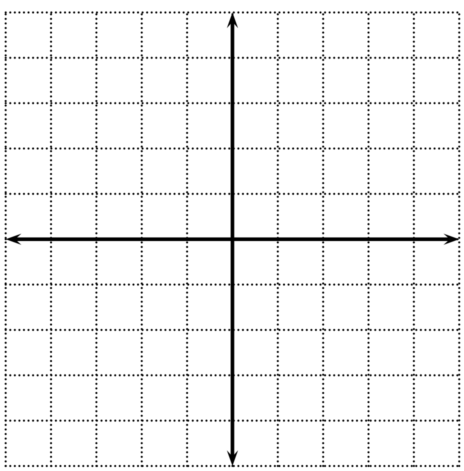
2. Horizontal intercepts:

3. Leading term:

4. Degree and end behavior:

5. Vertical intercept:

6. Sketch the graph using the information above and **test points** where needed.



Example 122. *(3.3: Example 9)* Answer the following for the polynomial function

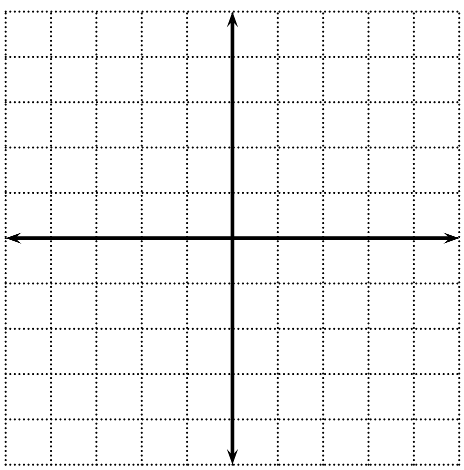
$$f(x) = x^4 - 4x^2 - 45$$

1. **Factor** the polynomial to find its horizontal intercepts:

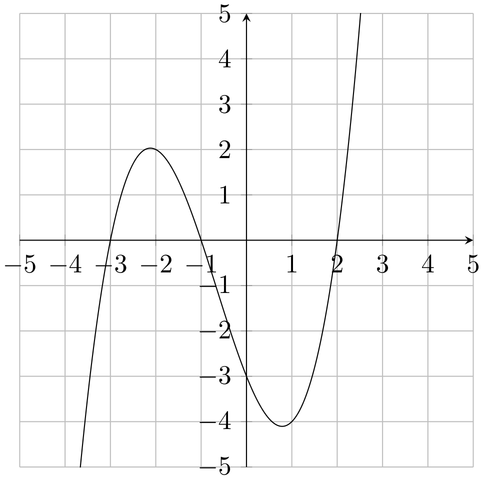
2. Leading term and end behavior:

3. Vertical intercept:

4. Sketch the graph using the information above and **test points** where needed.



Example 123. *(3.3: Try It #8)* What can we conclude about the equation for the polynomial function graphed below based on its intercepts, turning points and end behavior?



1. Degree:

2. Leading coefficient:

3. Factors:

4. Constant term:

Example 124. *(3.4: Example 2)* Answer the following for the polynomial function

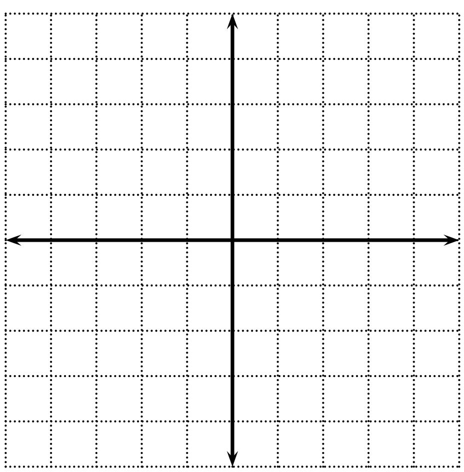
$$f(x) = x^6 - 3x^4 + 2x^2$$

1. **Factor** the polynomial to find its horizontal intercepts:

2. Leading term and end behavior:

3. Vertical intercept:

4. Sketch the graph using the information above and **test points** where needed.



Example 125. *(3.4: Example 4)* Answer the following for the polynomial function

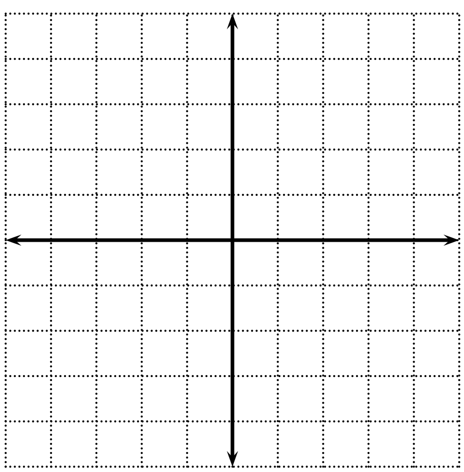
$$g(x) = (x-2)^2(2+3)$$

1. Horizontal intercepts:

2. Leading term and end behavior:

3. Vertical intercept:

4. Sketch the graph using the information above and **test points** where needed.



Looking at Examples 124 and 125 we notice:

Multiplicity:

Example 126. *(3.4: Try It #3* Answer the following for the polynomial function

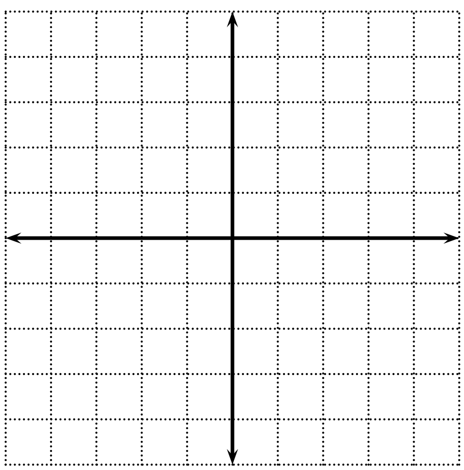
$$f(x) = \frac{1}{4}x(x-1)^4(x+3)^3$$

1. Real roots with multiplicity:

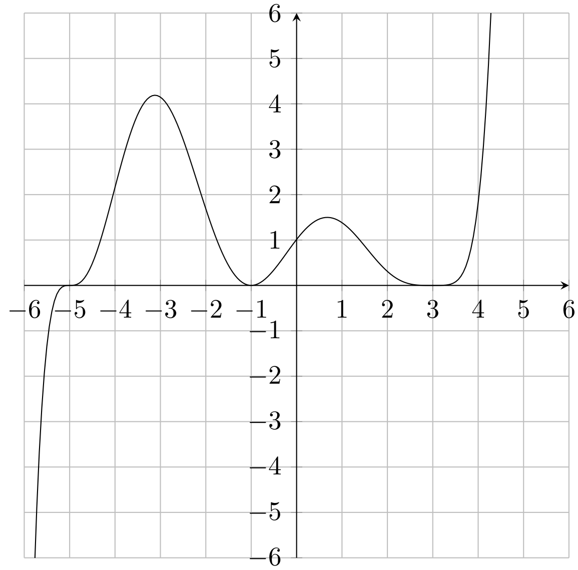
2. Leading term and end behavior:

3. Vertical intercept:

4. Sketch the graph using the information above and **test points** where needed.



Example 127. *(3.4: Try It #2)* What can we conclude about the equation for the polynomial function graphed below?



1. Constant term:
2. Degree even or odd?
3. Leading coefficient:
4. Real roots with multiplicity:
5. Turning points:
6. Possible equation:
7. Label all **local** and **global maxima** and **minima**.

GROUP WORK: PRACTICE GRAPHING POLYNOMIALS

Example 128. Answer the following for the polynomial function

$$f(x) = 2x^3 - 6x^2$$

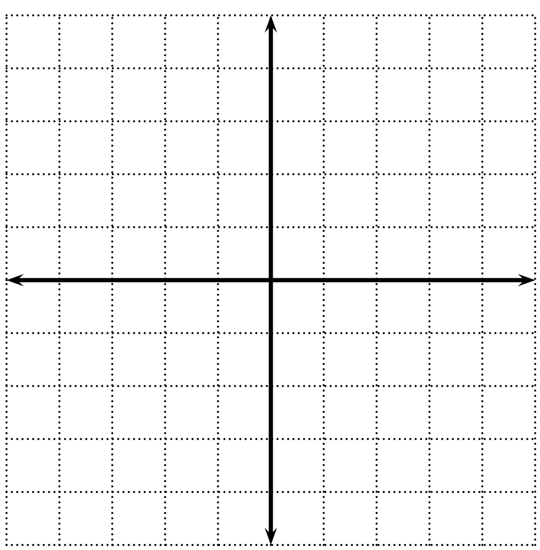
1. Factor the polynomial

2. Real roots with multiplicity:

3. Leading term and end behavior:

4. Vertical intercept:

5. Sketch the graph using the information above and **test points** where needed.



GROUP WORK: PRACTICE GRAPHING POLYNOMIALS

Example 129. Answer the following for the polynomial function

$$g(x) = -\frac{1}{4}x^4 + \frac{3}{2}x^3 - \frac{9}{4}x^2$$

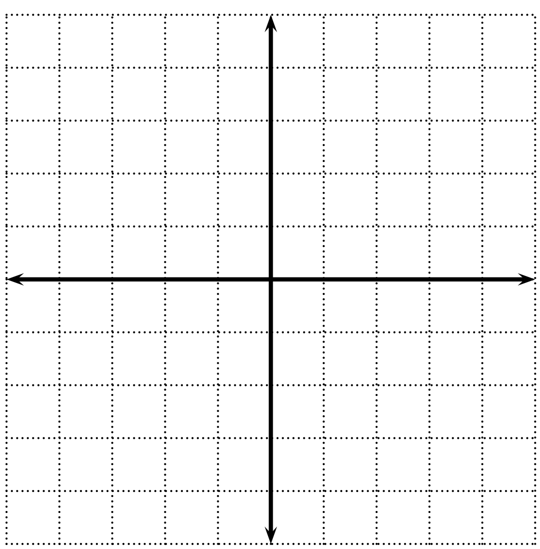
1. Factor the polynomial. *(Hint: Factor out a GCF of $-\frac{1}{4}$)*

2. Real roots with multiplicity:

3. Leading term and end behavior:

4. Vertical intercept:

5. Sketch the graph using the information above and **test points** where needed.



Chapter 5

Polynomial division, FTA and Rational Functions

5.1 3.5: Polynomial Division

Example 130. Divide $\dfrac{3x^2 + 19x + 28}{x + 4}$

Example 131. Divide $\dfrac{x^2 + 3x + 5}{x + 1}$

Example 132. Divide $(x^3 - 2x^2 - 9) \div (x - 3)$

Example 133. Divide $(-x^3 + 9x + 6x^4 - x^2 - 3) \div (1 + 3x)$

Example 134. Divide $\dfrac{5x^3 + 8x^2 - x + 6}{x + 2}$

GROUP WORK: PRACTICE ON POLYNOMIAL DIVISION

Use long division to simplify the following expressions.

1. $\dfrac{x^5 - 13x^4 - 120x + 80}{x+3}$

2. $\dfrac{x^3 - 729}{x-9}$

3. $\dfrac{-3x^4}{x+2}$

4. $\dfrac{5 - 3x + 2x^2 - x^3}{x+1}$

5.2 3.6: Fundamental Theorem of Algebra

Polynomial Factor Theorem:

So, the following statement are equivalent (i.e. they all mean the same thing):

1.

2.

3.

4.

General steps to factor a degree-n polynomial *(Compare to "How To..." in your book on p. 267.):*

1.

2.

3.

4.

5.

Example 135. *(Try It #2)* Find all real roots of $f(x) = x^3 + 4x^2 - 4x - 16$ given that $(x - 2)$ is a factor of the polynomial.

Rational Root Theorem:

Example 136. Answer the following for $f(x) = 2x^4 - 5x^3 + x^2 - 4$

1. *(Example 3)* Use the Rational Root Theorem to list all possible roots.

2. Check which of your answers $x = k$ from Part (a) are actually roots by evaluating $f(k)$.

3. Fully factor $f(x)$

Example 137. Answer the following for $g(x) = x^3 - 5x^2 + 2x + 1$

1. *(Try It #3)* Use the Rational Root Theorem to list all possible roots.

2. Fully factor $g(x)$

Example 138. Answer the following for the polynomial function

$$f(x) = x^3 + 2x^2 - 2x - 4$$

1. Use the Rational Root Theorem to list all possible roots.

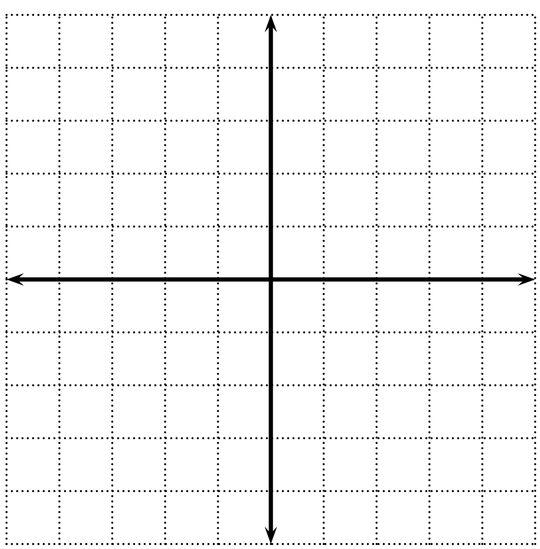
2. Fully factor $f(x)$

3. Real roots with multiplicity:

4. Leading term and end behavior:

5. Vertical intercept:

6. Sketch the graph using the information above and **test points** where needed.



Example 139. Answer the following for the polynomial function

$$g(x) = 2x^3 - 6x^2 + 8$$

1. Use the Rational Root Theorem to list all possible roots.

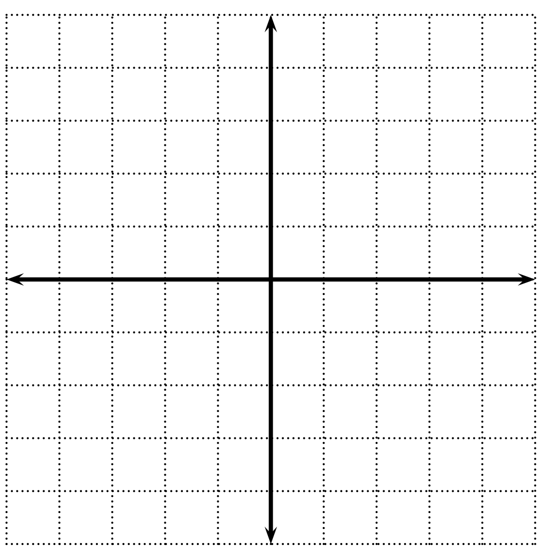
2. Fully factor $f(x)$

3. Real roots with multiplicity:

4. Leading term and end behavior:

5. Vertical intercept:

6. Sketch the graph using the information above and **test points** where needed.



Example 140. Answer the following for the polynomial function

$$h(x) = x^3 - x^2 + x - 1$$

1. Use the Rational Root Theorem to list all possible roots.

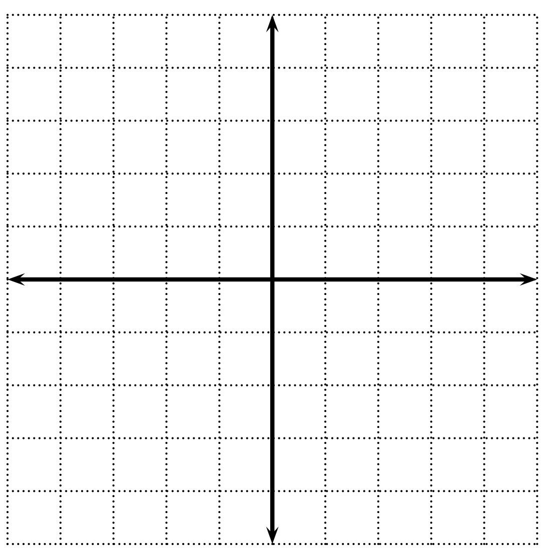
2. Fully factor $f(x)$

3. Real roots with multiplicity:

4. Leading term and end behavior:

5. Vertical intercept:

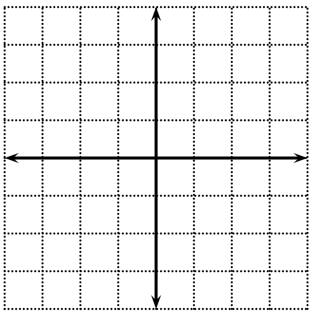
6. Sketch the graph using the information above and **test points** where needed.



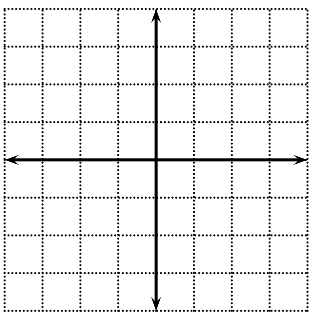
GROUP WORK: FINDING ROOTS

Example 141. Graph the following *quadratic* functions and find all real and complex roots, with multiplicity.

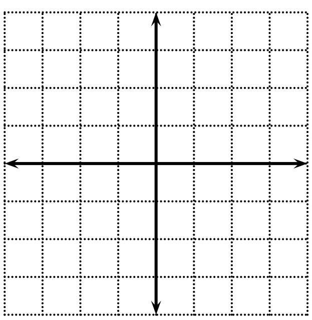
1. $f(x) = x^2 - 1$



2. $f(x) = x^2 + 2x + 1$



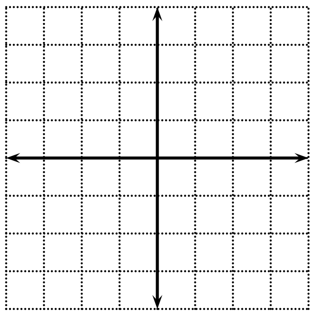
3. $f(x) = x^2 + 1$



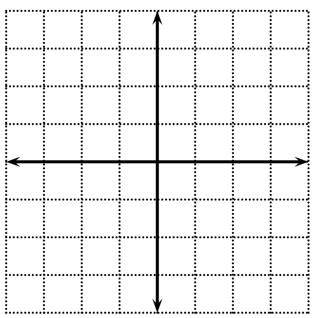
GROUP WORK: FINDING ROOTS

Example 142. Graph the following *cubic* functions and find all real and complex roots, with multiplicity.

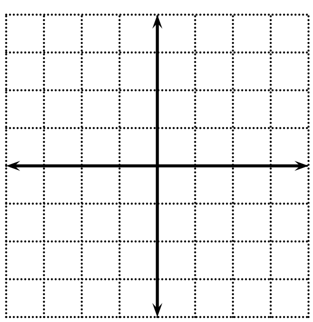
1. $f(x) = x^3$



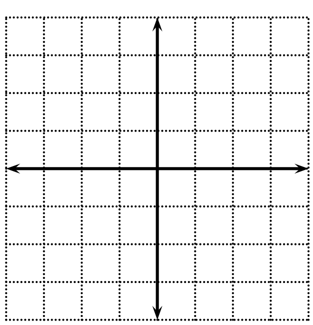
2. $f(x) = x^3 + 1$



3. $f(x) = -x^3 - 2x^2 - x - 2$ *(Hint: $(x = -2)$ is one root.)*



4. $f(x) = x^3 - x^2 - 8x + 12$ *(Hint: $(x = -3)$ is one root.)*



Fundamental Theorem of Algebra

Example 143. *(Try It #4)* Answer the following for $f(x) = 2x^3 + 5x^2 - 11x + 4$

1. Use the Rational Root Theorem and polynomial division to completely factor $f(x)$

2. Find all roots (including complex roots) of $f(x)$.

Complex Conjugate Theorem:

So complex roots of polynomial functions always come in ______________________

Example 144. Answer the following for $f(x) = x^3 - 3x^2 + 2x - 6$

1. Use the Rational Root Theorem and polynomial division to completely factor $f(x)$

2. Find all roots (including complex roots) of $f(x)$.

GROUP WORK: PRACTICE USING FTA

Example 145. For each of the given polynomial functions, use the Rational Root Theorem, polynomial division and the Fundamental Theorem of Algebra to completely factor the function. Then find all roots, real and complex.

1. $f(x) = 9x^4 - 12x^2$ Factored form: $f(x) =$ ____________________

   Roots: ________________

2. $g(x) = x^3 - 5x^2 - 2x + 10$ Factored form: $g(x) =$ ____________________

   Roots: ________________

3. $h(x) = x^3 - 5x^2 + 2x + 8$ Factored form: $h(x) =$ ____________________

   Roots: ________________

GROUP WORK: PRACTICE USING FTA

Example 146. For each of the following equations, find *all* solutions and identify them as rational, real (not rational) and complex (nonreal).

1. $10x^3 - 15x^2 - 16x + 12 = 0$

2. $x^3 + 2x^2 + 6x - 4 = 0$

3. $2x^3 - 3x^2 + 2x - 6 = 0$

5.3 3.7: Rational Functions

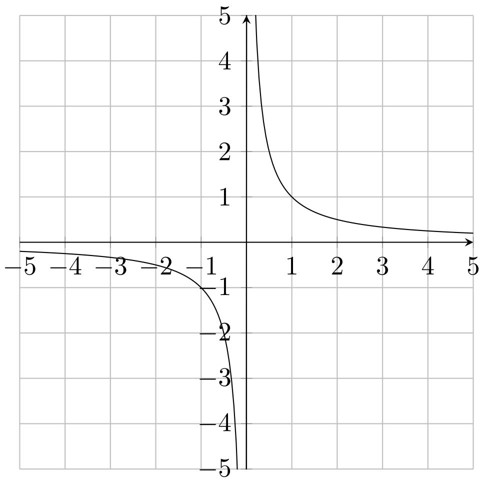
Definitions:

- Rational Function:

- Vertical Asymptote:

- Horizontal Asymptote:

Example 147. $f(x) = \dfrac{1}{x}$

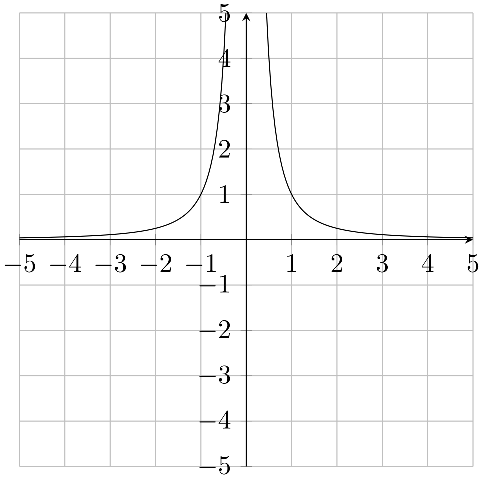


- Horizontal asymptote:

- Vertical asymptote:

Asymptotes are always ____

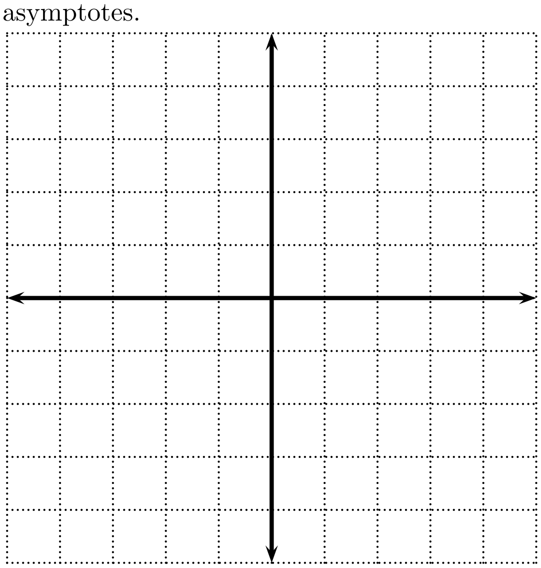
Example 148. *(Try It #1)* $f(x) = \dfrac{1}{x^2}$



- Horizontal asymptote:

- Vertical asymptote:

Example 149. *(Try It #2)* Sketch a graph of the rational function $g(x) = \dfrac{1}{(x-3)^2} - 4$ and find its vertical and horizontal asymptotes.



Domains of Rational Functions

Example 150. Find the domain of $f(x) = \dfrac{3x}{x-1}$

Example 151. *(Try It #4)* Find the domain of $F(x) = \dfrac{4x}{5x^2 - 30x + 25}$

Example 152. *(Try It #5)* Find the domain of $G(x) = \dfrac{x^2 - 25}{x^3 - 6x^2 + 5x}$

Let's completely factor (numerator and denominator) $F(x)$ and $G(x)$ in Examples 151 and 152.

Understanding the domain of a rational function:

- A factor of the denominator that **cannot cancel** with any factor of the numerator gives a ____________________________ in the graph.

- A factor of the denominator that **can cancel** with a factor of the numerator gives a ______________________________

  in the graph called a ______________________________.

Horizontal Asymptotes

- If deg N $<$ deg D:

- If deg N $=$ deg D:

- If deg N $=$ (deg D)+1:

- If deg N $>$ (deg D)+1:

Example 153. For each of the following functions, find any vertical and horizontal asymptotes:

1. $f(x) = \dfrac{2x+1}{x+1}$

2. $g(x) = \dfrac{4}{x^2+1}$

3. $h(x) = \dfrac{2}{(x-1)^2}$

4. $f(x) = \dfrac{x^2+x-2}{x^2-x-6}$

5. $g(x) = \dfrac{2x^2}{x^2+1}$

6. *(Try It #6)* $h(x) = \dfrac{(2x-1)(2x+1)}{(x-2)(x+3)}$

Graphing Rational Functions

Example 154. Find the following for $G(x) = \dfrac{x^2 - 25}{x^3 - 6x^2 + 5x}$ from Example 152.

1. Vertical intercept

2. Horizontal asymptote:

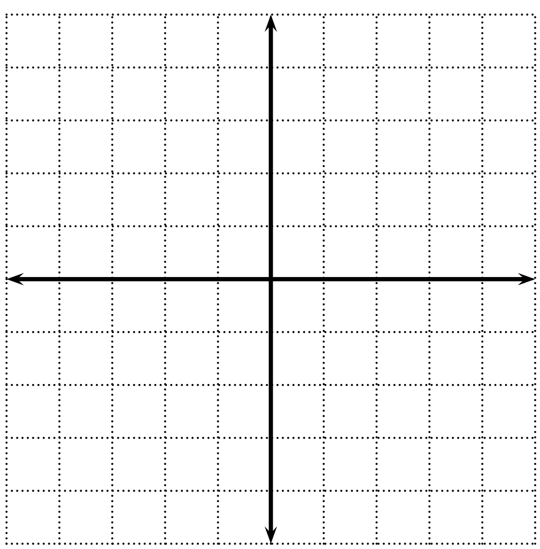
3. Write $G(x)$ in factored form.

4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

7. Sketch the graph, using extra test points if necessary



Example 155. *(Try It #8)* Find the following for $f(x) = \dfrac{(x+2)^2(x-2)}{2(x-1)^2(x-3)}$.

1. Vertical intercept

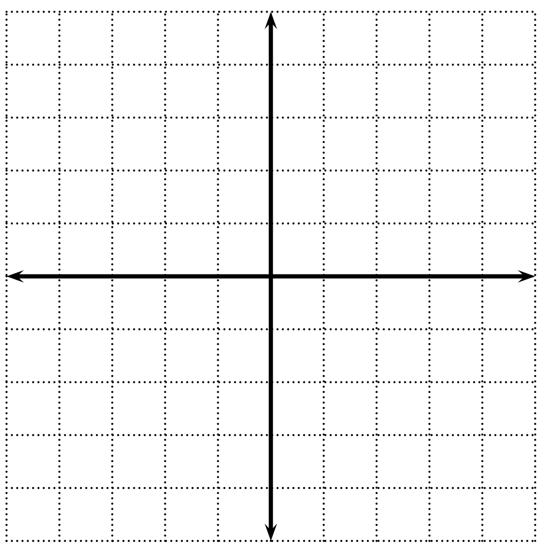
2. Horizontal asymptote:

3. Write $f(x)$ in factored form. *Already done!*
4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

7. Sketch the graph, using extra test points if necessary



GROUP WORK: GRAPHING RATIONAL FUNCTIONS

Example 156. Find the following for $f(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6}$ from Example 153.

1. Vertical intercept

2. Horizontal asymptote:

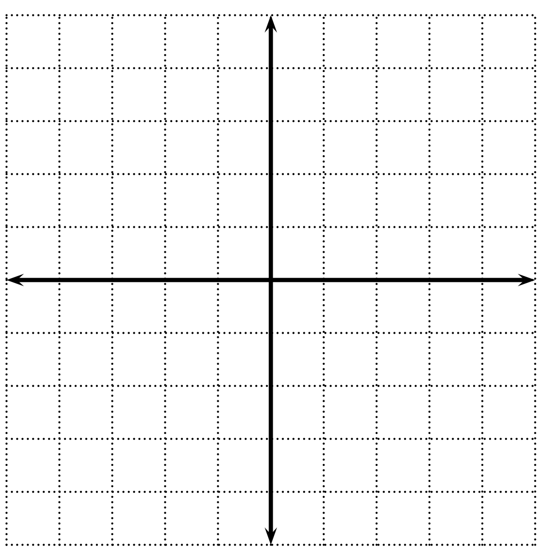
3. Write $f(x)$ in factored form.

4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

7. Sketch the graph, using extra test points if necessary



GROUP WORK: GRAPHING RATIONAL FUNCTIONS

Example 157. Find the following for $g(x) = \dfrac{3x}{x^2 + x - 2}$.

1. Vertical intercept

2. Horizontal asymptote:

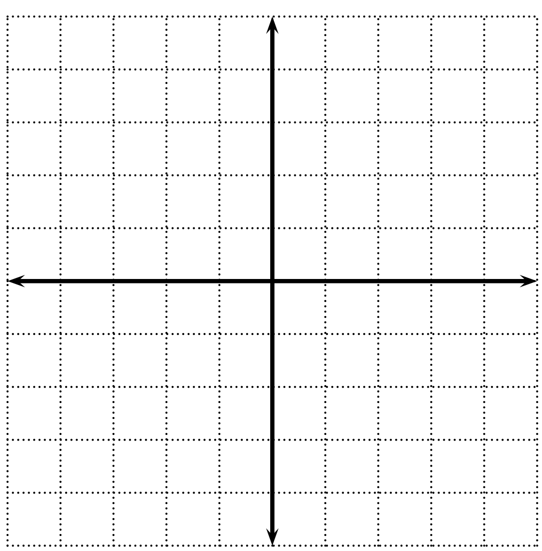
3. Write $g(x)$ in factored form.

4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

7. Sketch the graph, using extra test points if necessary



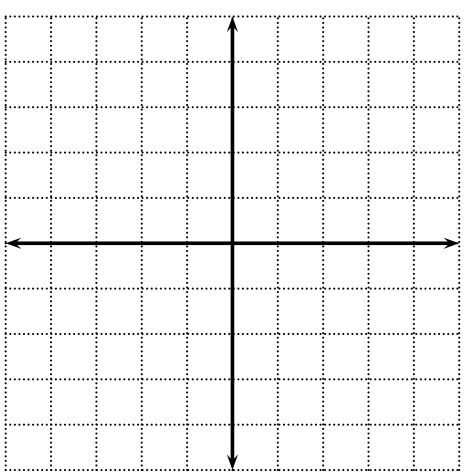
Slant Asymptotes

Recall: If deg N = (deg D)+1 then the graph has a **slant asymptote**.

Example 158. $f(x) = \dfrac{x^2 - x}{x + 1}$

Example 159. Find the following for $g(x) = \dfrac{3x^2 + 1}{x}$.

1. Vertical intercept
2. Horizontal or slant asymptote:
3. Write $g(x)$ in factored form.
4. Vertical asymptote(s):
5. Holes:
6. Horizontal intercepts:
7. Sketch the graph, using extra test points if necessary



Example 160. Find the following for $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2}$.

1. Vertical intercept

2. Horizontal or slant asymptote:

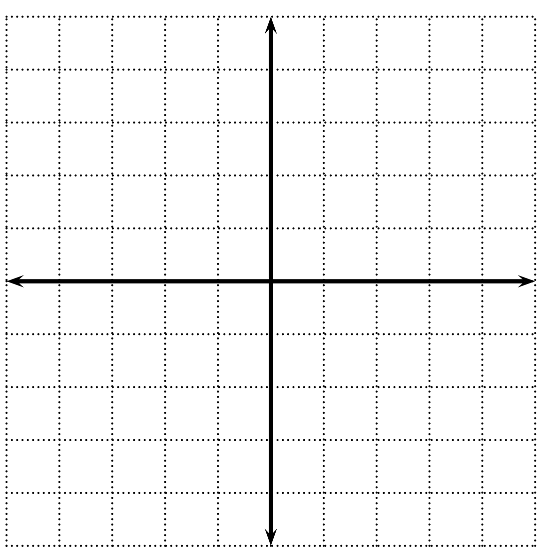
3. Write $g(x)$ in factored form.

4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

7. Sketch the graph, using extra test points if necessary



MORE PRACTICE GRAPHING RATIONAL FUNCTIONS

Example 161. Find the following for $f(x) = \dfrac{x^3}{2x^2 - 8}$.

1. Vertical intercept

2. Horizontal or slant asymptote:

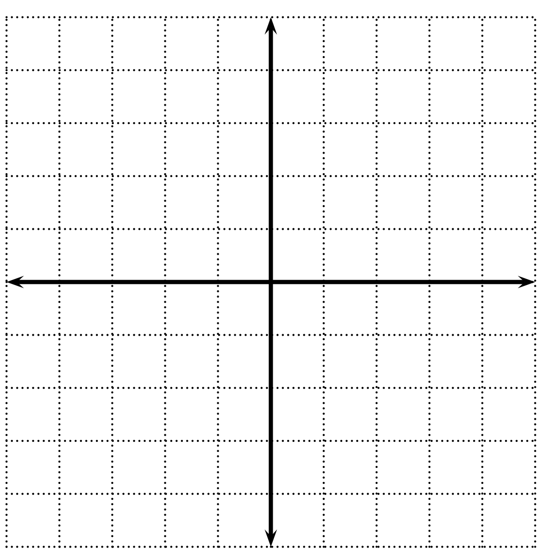
3. Write $g(x)$ in factored form.

4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

7. Sketch the graph, using extra test points if necessary. Choose an appropriate scale for your axes.



MORE PRACTICE GRAPHING RATIONAL FUNCTIONS

Example 162. Find the following for $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$.

1. Vertical intercept

2. Horizontal or slant asymptote:

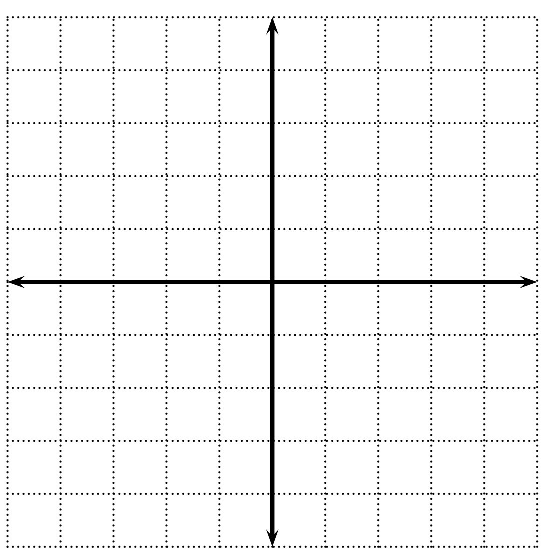
3. Write $g(x)$ in factored form.

4. Vertical asymptote(s):

5. Holes:

6. Horizontal intercepts:

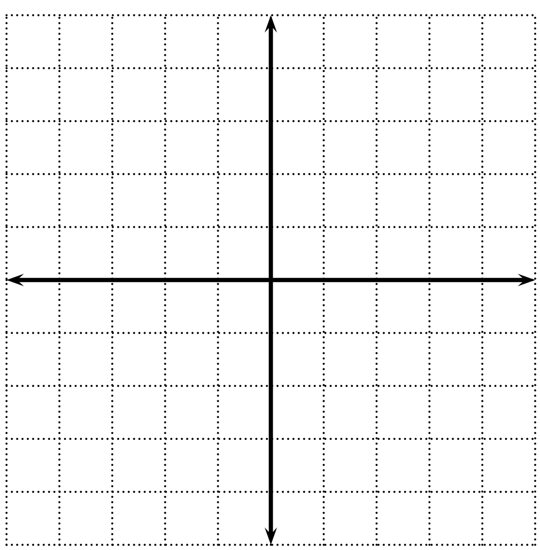
7. Sketch the graph, using extra test points if necessary. Choose an appropriate scale for your axes.



MORE PRACTICE GRAPHING RATIONAL FUNCTIONS

Example 163. Find the following for $f(x) = \dfrac{6x^2 - 11x + 3}{6x^2 - 3x - 3}$.

1. Vertical intercept
2. Horizontal or slant asymptote:
3. Write $g(x)$ in factored form.
4. Vertical asymptote(s):
5. Holes:
6. Horizontal intercepts:
7. Sketch the graph, using extra test points if necessary.



5.4 Solving Nonlinear Equations and Inequalities

Solving Rational Equations

Example 164. Solve $\frac{4x}{9} - \frac{1}{3} = x + \frac{5}{3}$

Example 165. Solve $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$

Solving Radical Equations

Example 166. Solve $x = \sqrt{-4 + 4x}$

Example 167. Solve $4 = \sqrt{-3x + 10} = x$

Solving Polynomial Inequalities

Example 168. Solve $x^2 - x - 20 < 0$

Example 169. Solve $3x^3 - x^2 - 12x > -4$

Solving Rational Inequalities

Example 170. Solve $\dfrac{2x-7}{x-5} \leq 3$

Example 171. Solve $\dfrac{5}{x-6} > \dfrac{3}{x+2}$

HOMEWORK

Example 172. Solve $2x^2 + 3x \leq 5$

Example 173. Solve $-3 + \sqrt{x + 59} = x$

Example 174. Solve $x^2 + 16x + 24 > 6x$

Example 175. Solve $\dfrac{3x}{x - 4} = 5 + \dfrac{12}{x - 4}$

HOMEWORK

Example 176. Solve $\dfrac{1}{6x^2} = \dfrac{1}{3x^2} - \dfrac{1}{x}$

Example 177. Solve $\dfrac{x-2}{x-3} \geq -3$

Example 178. Solve $-\dfrac{3}{x+7} \leq -\dfrac{4}{x+8}$

Chapter 6

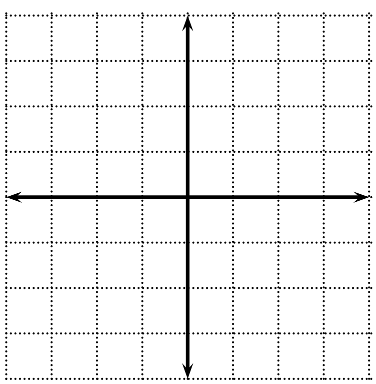
Radicals and Variation

6.1 3.8: Inverses and Radical Functions

Recall:

- A function must be ____________________ (i.e. pass the ____________________________) to have an inverse.
- If it isn't, we must ____

Example 179. *(Try It #3)* Find the inverse of the function $f(x) = x^2 + 1$, restricting the domain if necessary. Sketch a graph of both $f(x)$ and $f^{-1}(x)$ on the same axes, and state the domain and range of both functions.

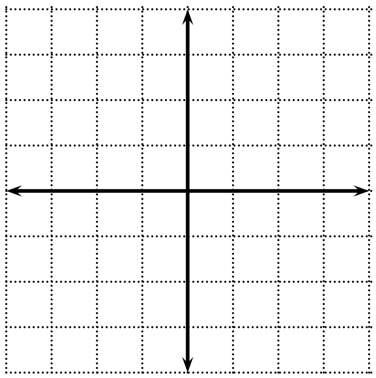


$f^{-1}(x) =$ ____________________________

Domain of $f(x)$: _________________ Domain of $f^{-1}(x)$: _________________

Range of $f(x)$: _________________ Range of $f^{-1}(x)$: _________________

Example 180. *(Try It #4)* Find the inverse of the function $f(x) = \sqrt{2x+3}$, restricting the domain if necessary. Sketch a graph of both $f(x)$ and $f^{-1}(x)$ on the same axes, and state the domain and range of both functions.



$f^{-1}(x) =$ ______________________

Domain of $f(x)$: ______________ Domain of $f^{-1}(x)$: ______________

Range of $f(x)$: ______________ Range of $f^{-1}(x)$: ______________

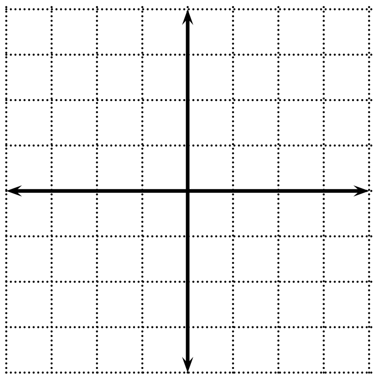
Example 181. *(Try It #5)* Find the inverse of the function $f(x) = \dfrac{x+3}{x-2}$. Then state the domain and range of both functions.

$f^{-1}(x) =$ ______________________

Domain of $f(x)$: ______________ Domain of $f^{-1}(x)$: ______________

Range of $f(x)$: ______________ Range of $f^{-1}(x)$: ______________

Example 182. Find the inverse of the function $f(x) = 4 - 2x^3$, restricting the domain if necessary. Sketch a graph of both $f(x)$ and $f^{-1}(x)$ on the same axes, and state the domain and range of both functions.



$f^{-1}(x) =$ ____________________________

Domain of $f(x)$: _________________ Domain of $f^{-1}(x)$: _________________

Range of $f(x)$: _________________ Range of $f^{-1}(x)$: _________________

Example 183. *(HW #61)* A container holds 100 mL of a solution that is 25 mL acid if n ml of a solution that is 60% acid is added, then the function $C(n) = \dfrac{25 + 0.6n}{100 + n}$ gives the concentration, C, of the solution as a function of n. Express n as a function of C, and determine the number of mL that need to be added to have a solution that is 50% acid.

6.2 Direct and Inverse Variation

Direct Variation:

Example 184. *(Try It #1* The quantity y varies directly with the square of x. If $y = 24$ when $x = 3$, find y when $x = 4$.

- Write the model equation.
- Plug in the give quantities to solve for k.
- Write the variation equation with k known.
- Plug in x and solve for y.

Example 185. The quantity y varies directly with the cube of x. If $x = 3$, then $y = 5$. Find y when $x = 4$.

- Write the model equation.
- Plug in the give quantities to solve for k.
- Write the variation equation with k known.
- Plug in x and solve for y.

<u>Inverse Variation</u>:

Example 186. *(Try It #2* The quantity y varies inversely with the square of x. If $y = 8$ when $x = 3$, find y when $x = 4$.

- Write the model equation.
- Plug in the give quantities to solve for k.
- Write the variation equation with k known.
- Plug in x and solve for y.

Example 187. The quantity y varies inversely with x. If $x = 8$, then $y = 12$. Find y when $x = 36$.

- Write the model equation.
- Plug in the give quantities to solve for k.
- Write the variation equation with k known.
- Plug in x and solve for y.

Chapter 7

Conic Sections

7.1 10.1A: Geometry of Ellipses

EXPLORATION OF ELLIPSES
Group work

Circles

1. Use a loop of string, a pushpin and a pen to draw a circle.
2. Use your technique to draw a circle with radius of 3 inches.
   (a) How long does the string need to be to draw this circle (ignore the "tails" leftover from tying the knot)?

   (b) In general, how long would the string need to be to draw a circle with radius r inches?
3. On graph paper, use your technique to draw a circle with radius 5 units and center $(3, -2)$. Where did you place your pushpin?

Ellipses

1. Use a loop of string, two pushpins and a pen to draw an ellipse.
2. On graph paper, use your technique to draw an ellipse centered at $(0, 0)$ so that the longest part of the ellipse is along the horizontal axis. *(Tip: Your work will be easier later if your string length is an even number of graph paper units.)*
   (a) Where did you place your pushpins?

   (b) Can you draw another ellipse centered at $(0, 0)$ with the longest part of the ellipse is along the horizontal axis? Where did you place the pushpins for this ellipse?

   (c) Draw one more ellipse centered at $(0, 0)$ with the longest part of the ellipse is along the horizontal axis. Where did you place the pushpins this time?

   (d) The points on the ellipse that are farthest from the center are called **vertices**. What are the vertices of each of the three ellipses your just drew? (Each ellipse has two vertices, and for these three ellipses they should be on the horizontal axis.)

Ellipse (a) vertices	Ellipse (b) vertices	Ellipse (c) vertices

(e) What was the length of string you used for each of the three ellipses your just drew? (Even if they are all the same length, record them below.)

Ellipse (a) string	Ellipse (b) string	Ellipse (c) string

(f) The location of the pushpins are the **foci** of the ellipse. What pattern do you notice between the foci, vertices and string length for each of the three ellipses your just drew?

3. On graph paper, use your technique to draw an ellipse centered at $(0, 0)$ with one vertex at $(0, 5)$.

(a) Where is the other vertex?

(b) Where are the foci?

(c) What is your string length?

(d) Can you draw another ellipse centered at $(0, 0)$ with one vertex at $(0, 5)$? Where are the foci? What is the string length?

4. On graph paper, use your technique to draw an ellipse with one focus at $(3, 2)$ and one vertex at $(7, 2)$.

(a) Where is the other focus?

(b) Where is the center?

(c) What is your string length?

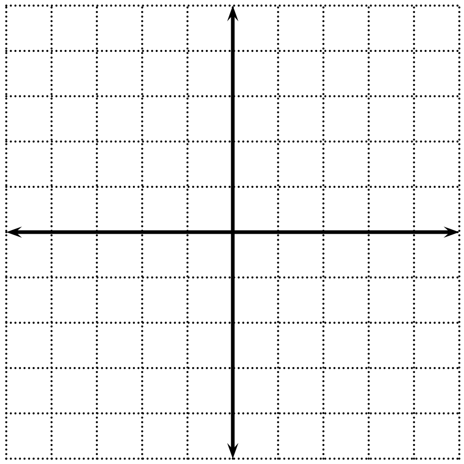
(d) If the center of your ellipse is between $(3, 2)$ and $(7, 2)$, draw another ellipse with one focus at $(3, 2)$ and one vertex at $(7, 2)$ but with the center not between $(3, 2)$ and $(7, 2)$. Likewise, if the center of your ellipse is not between $(3, 2)$ and $(7, 2)$, draw another ellipse with one focus at $(3, 2)$ and one vertex at $(7, 2)$ but with the center between $(3, 2)$ and $(7, 2)$. List the center, other focus and string length for this ellipse.

7.2 10.1B: Algebra of Ellipses

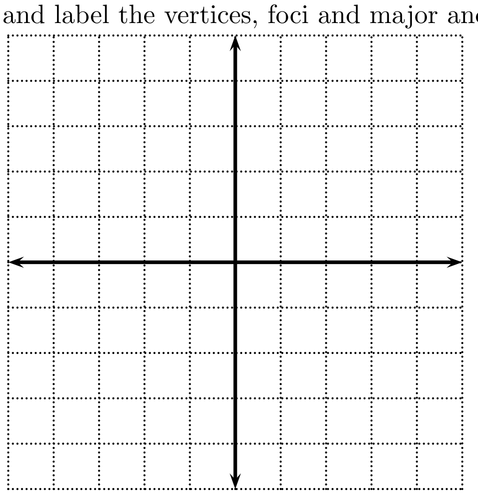
Vocabulary

The **standard equation** of an ellipse with **center at** $(0, 0)$ is:

Example 188. *(Try It #1)* Find an equation for the ellipse with vertices $(0, \pm 4)$ and foci $(0, \pm\sqrt{15})$. Then sketch a graph of the ellipse and label the vertices, foci and major and minor axes.

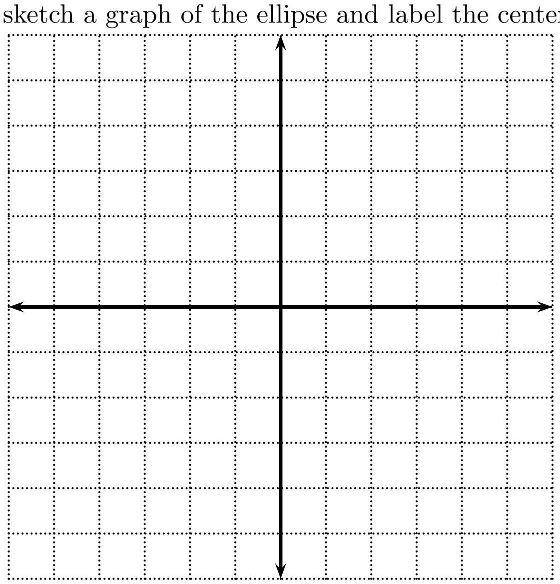


Example 189. Find an equation for the ellipse with foci $(\pm 3, 0)$ and vertices $(\pm 4, 0)$. Then sketch a graph of the ellipse and label the vertices, foci and major and minor axes.



The **standard equation** of an ellipse with **center at** (h, k) is:

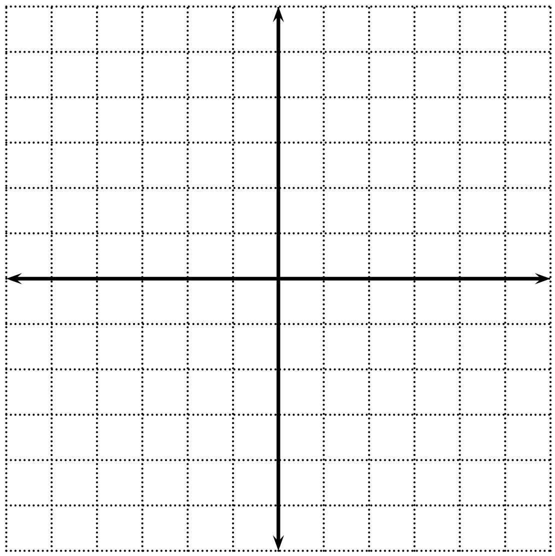
Example 190. *(Try It #2)* Find an equation for the ellipse with vertices $(-3, 3)$ and $(5, 3)$ and foci $(1 \pm 2\sqrt{3}, 3)$. Then sketch a graph of the ellipse and label the center, vertices, foci and major and minor axes.



Example 191. *(Try It #3)* Sketch a graph of the ellipse given by the equation

$$\frac{x^2}{36} + \frac{y^2}{4} = 1$$

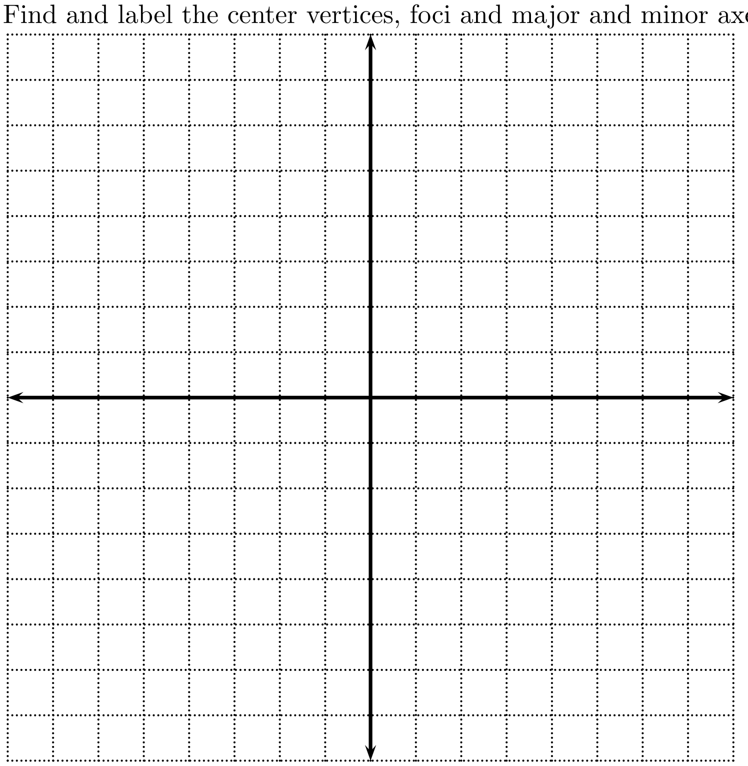
Find and label the center vertices, foci and major and minor axes.



Example 192. *(Try It #4)* Sketch a graph of the ellipse given by the equation

$$49x^2 + 16y^2 = 784$$

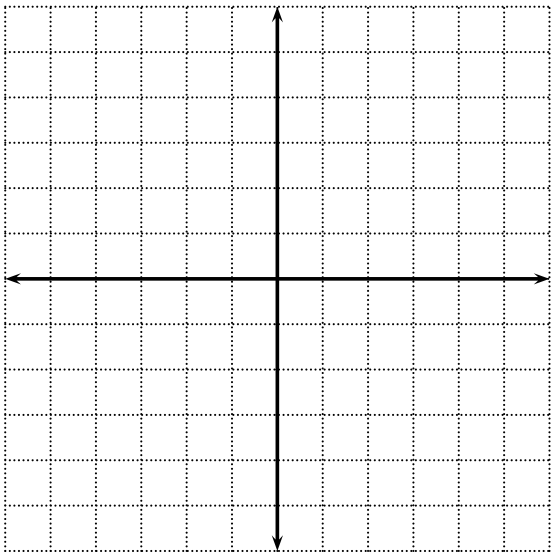
Find and label the center vertices, foci and major and minor axes.



Example 193. *(Try It #5)* Sketch a graph of the ellipse given by the equation

$$\frac{(x-4)^2}{36} + \frac{(y-2)^2}{20} = 1$$

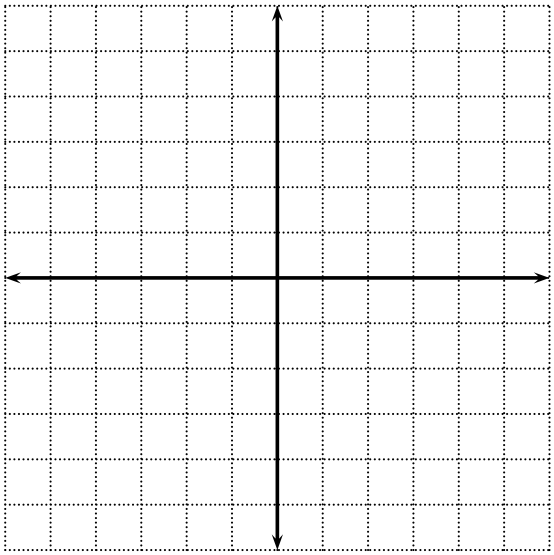
Find and label the center vertices, foci and major and minor axes. (Choose an appropriate scale for the x and y axes.)



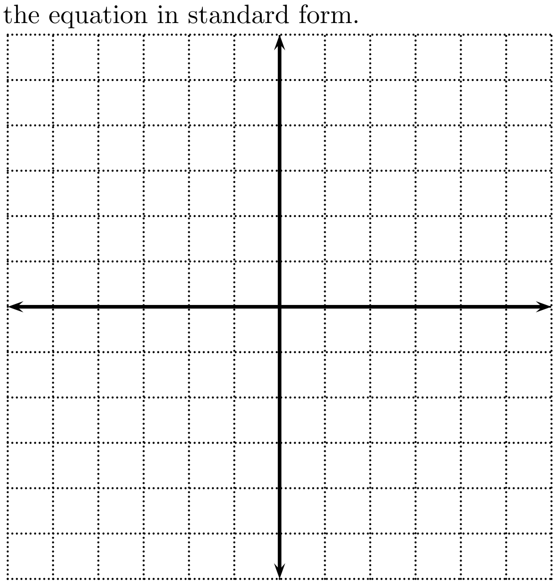
Completing the square

Example 194. Rewrite the equation $9x^2 + 4y^2 + 36x - 8y + 4 = 0$ in standard form.

Example 195. *(Try It #6)* Find the center, vertices and foci and sketch the graph of $4x^2 + y^2 - 24x + 2y + 21 = 0$ by first writing the equation in standard form.



Example 196. Find the center, vertices and foci and sketch the graph of $5x^2 + 9y^2 + 10x - 54y + 41 = 0$ by first writing the equation in standard form.



7.3 10.2: Hyperbolas

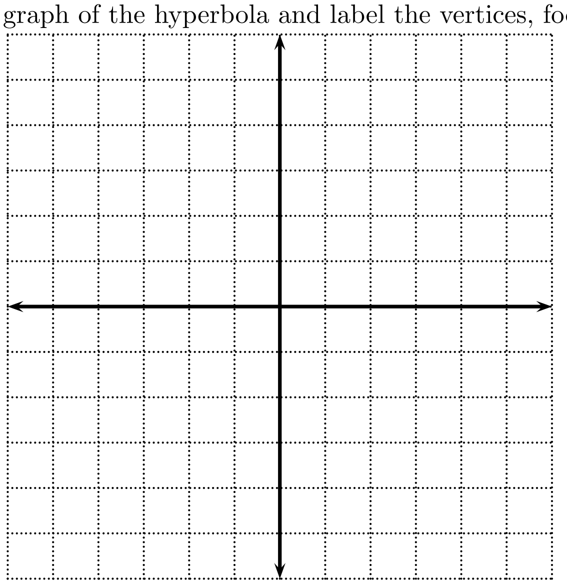
Vocabulary

The points on an **ellipse** have a ____ of distances from 2 foci.

The points on a **hyperbola** have a ____ of distances from 2 foci.

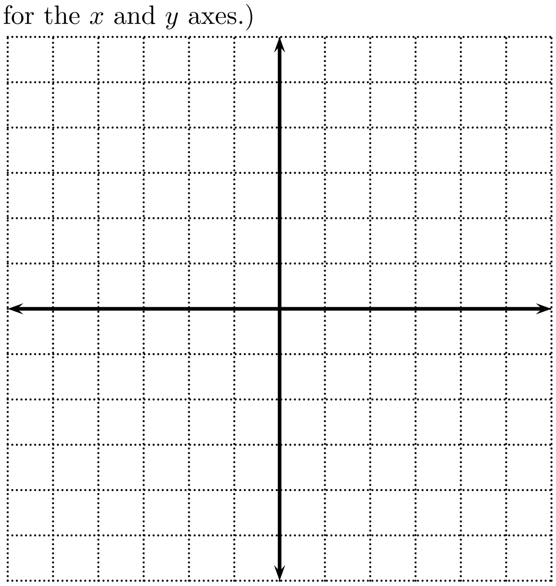
The **standard equations** for a hyperbola with **center** $(0, 0)$ are:

Example 197. *(Try It #2)* Find an equation for the hyperbola with vertices $(0, \pm 2)$ and foci $(0, \pm 2\sqrt{5})$. Then sketch a graph of the hyperbola and label the vertices, foci and asymptotes.

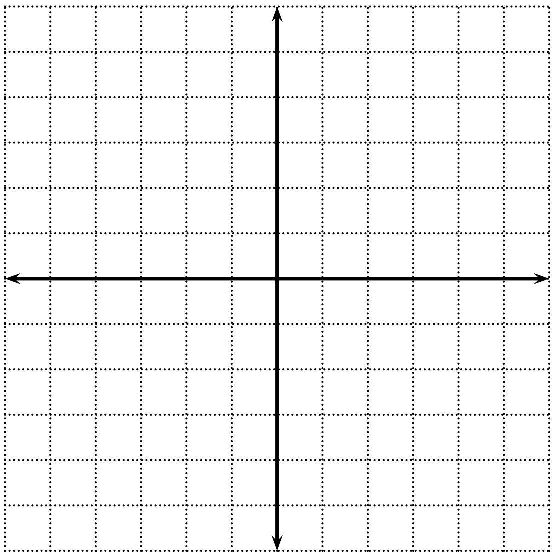


The **standard equations** for a hyperbola with **center** (h, k) are:

Example 198. *(Try It #3)* Find an equation for the hyperbola with vertices $(1, -2)$ and $(1, 8)$ and foci $(1, -10)$ and $(1, 16)$. Then sketch a graph of the hyperbola and label the vertices, foci and asymptotes. (Choose an appropriate scale for the x and y axes.)



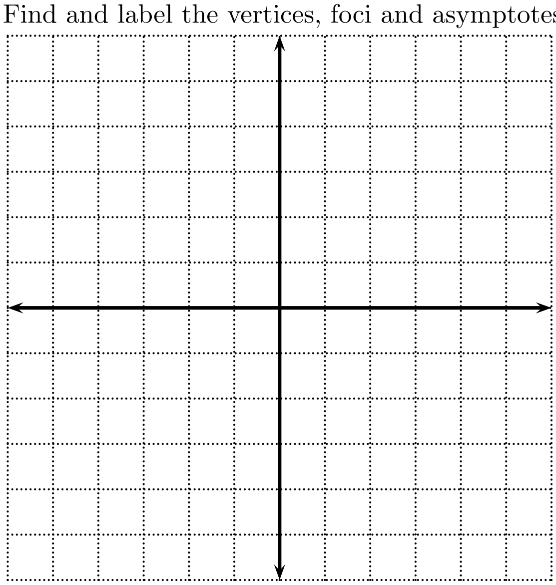
Example 199. Find an equation for the hyperbola with vertices $(12, 1)$ and $(2, 1)$ and foci $(-3, 1)$ and $(3, 1)$. Then sketch a graph of the hyperbola and label the vertices, foci and asymptotes. (Choose an appropriate scale for the x and y axes.)



Example 200. *(Try It #4)* Sketch a graph of the hyperbola given by the equation

$$\frac{x^2}{144} - \frac{y^2}{81} = 1$$

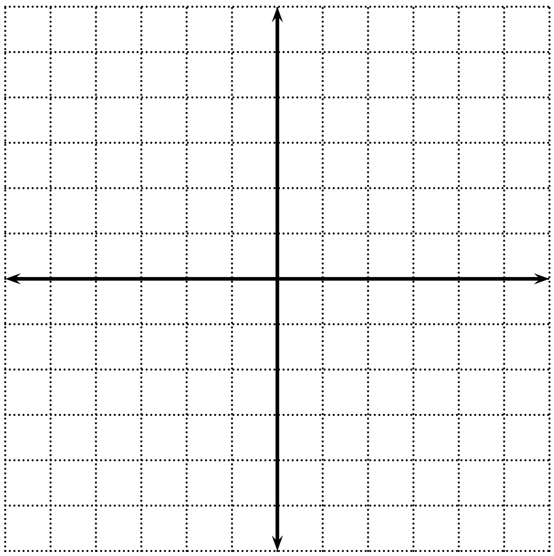
Find and label the vertices, foci and asymptotes. (Choose an appropriate scale for the x and y axes.)



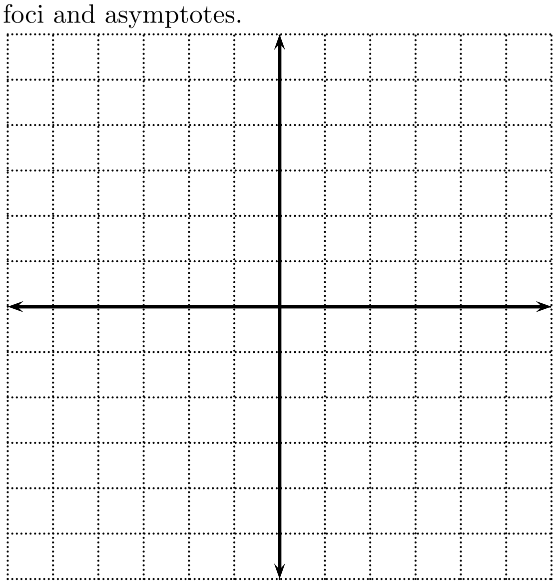
Example 201. *(Try It #5)* Sketch a graph of the hyperbola given by the equation

$$\frac{(y+4)^2}{100} - \frac{(x-3)^2}{64} = 1$$

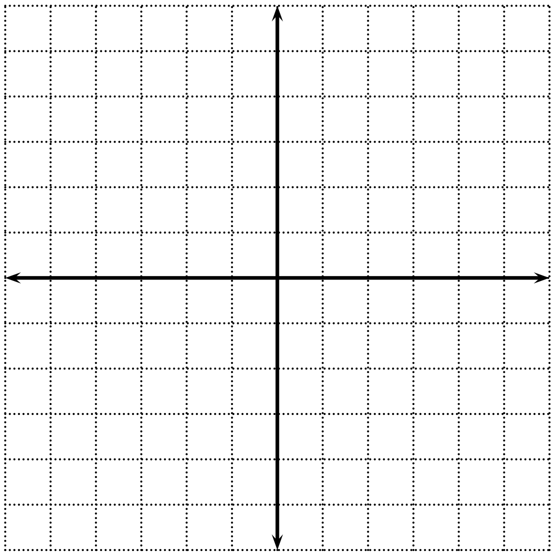
Find and label the center, vertices, foci and asymptotes. (Choose an appropriate scale for the x and y axes.)



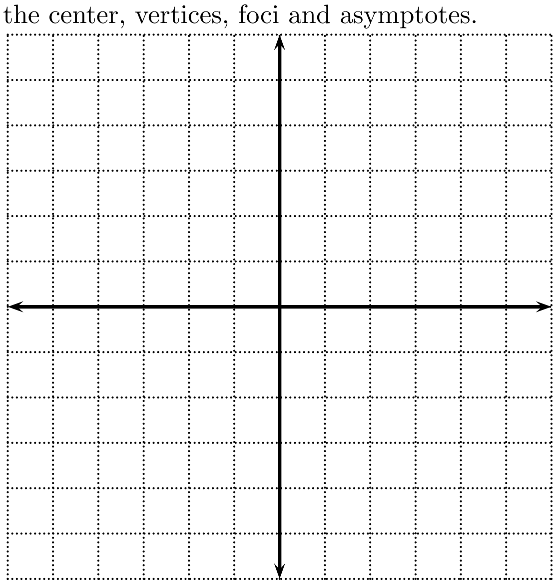
Example 202. Sketch a graph of the hyperbola given by the equation $4y^2 - 9x^2 = 36$. Find and label the center, vertices, foci and asymptotes.



Example 203. Sketch a graph of the hyperbola given by the equation $9x^2 - 4y^2 + 8y - 40 = 0$. Find and label the center, vertices, foci and asymptotes.



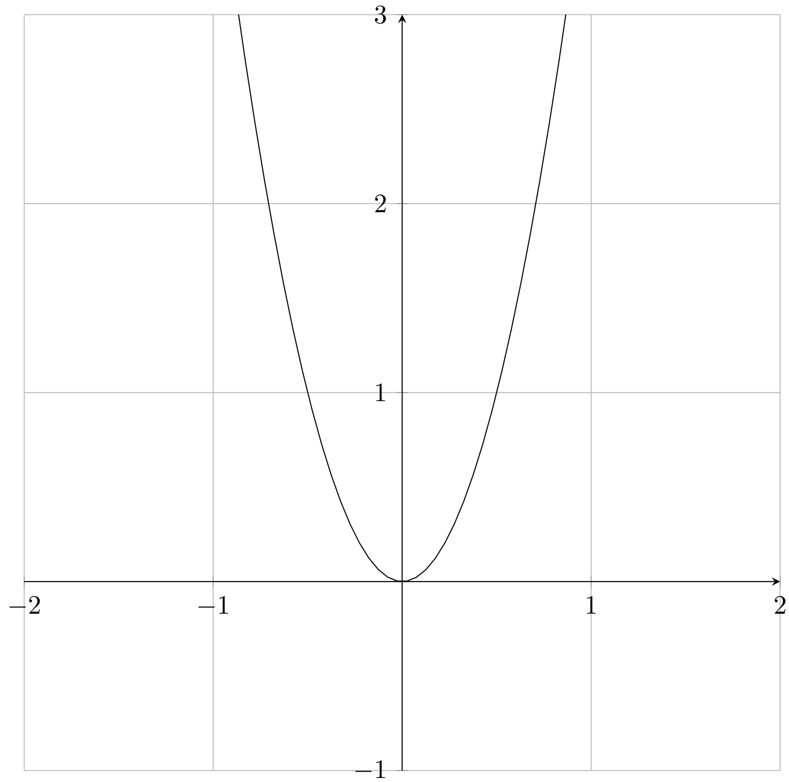
Example 204. Sketch a graph of the hyperbola given by the equation $-x^2 + 8x + 4y^2 - 40y + 88 = 0$. Find and label the center, vertices, foci and asymptotes.



7.4 10.3-10.4: Parabolas and Conic Sections

A **parabola** can be defined as:

Example 205. $y = 4x^2$



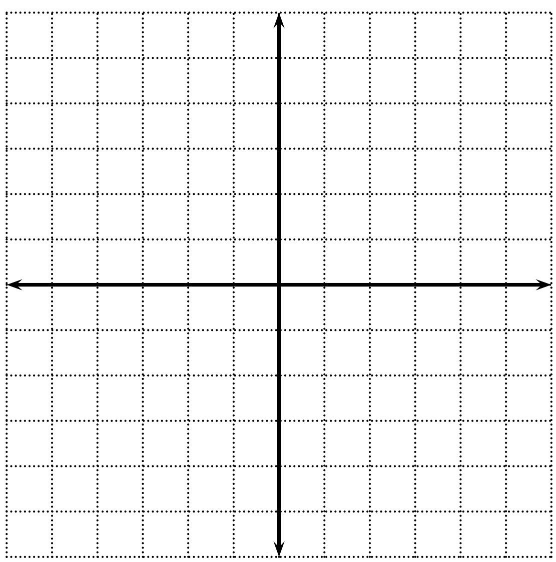
Not all parabolas are functions!

Standard Conic Form of a Parabola: Case 1

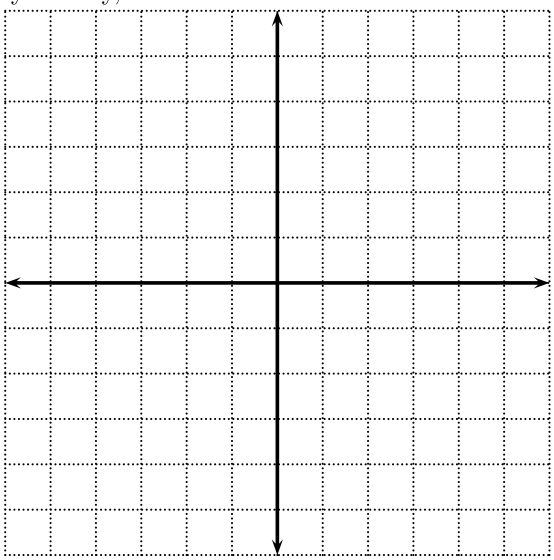
Compare to **vertex (standard) form**:

Example 206. *(10.3: Try It #5)* Find the following for $(x+2)^2 = -20(y-3)$

- Find and graph the vertex:
- Find p:
- Find and graph the focus:
- Find and graph the directrix:
- Find and graph the axis of symmetry:
- Use $2p$ and the axis of symmetry to find two more points on the parabola.
- Sketch the graph. (Choose an appropriate scale for the x and y axes.)



Example 207. Us the steps in Example 206 to sketch a graph of $(x-3)^2 = 8(y-2)$. Find and label the vertex, axis of symmetry, focus and directrix.



<u>Standard Conic Form of a Parabola: Case 2</u>

Steps for graphing a parabola given an equation in standard conic form

1.

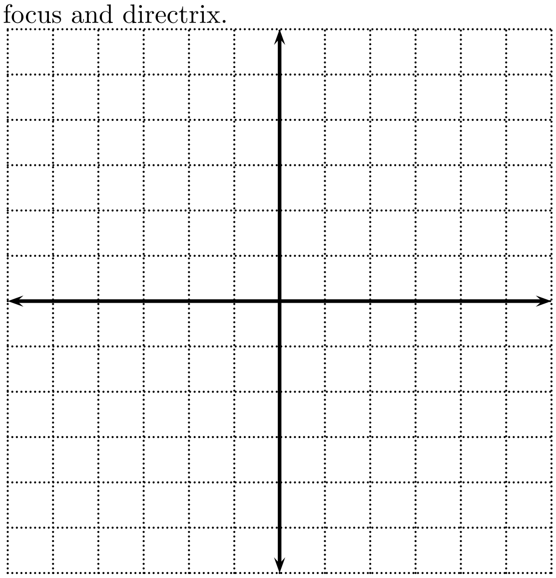
2.

3.

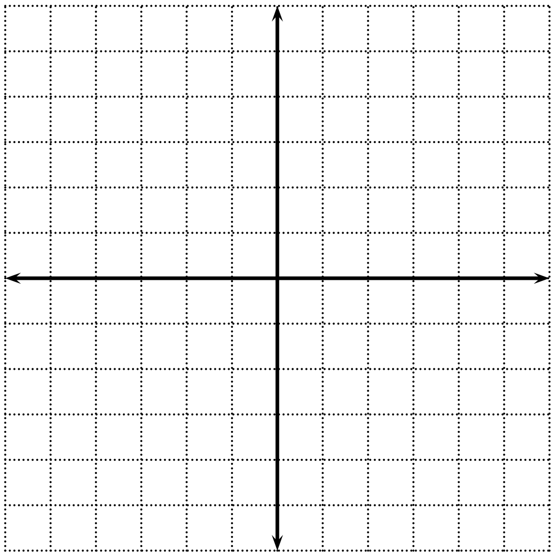
4.

5.

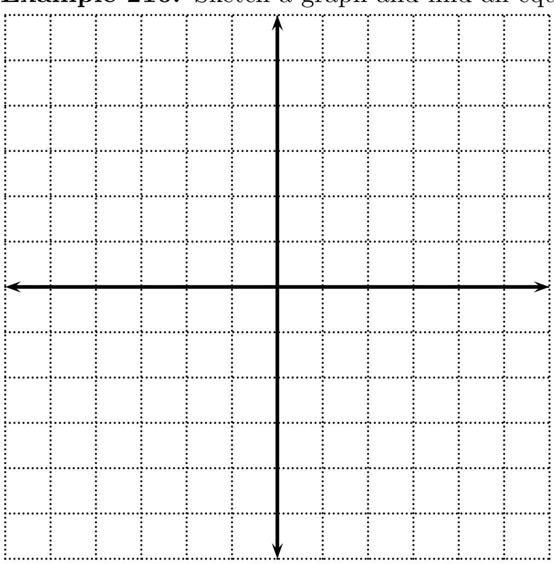
Example 208. *(10.3: Try It #4)* Sketch a graph of $(y+1)^2 = 4(x-8)$. Find and label the vertex, axis of symmetry, focus and directrix.



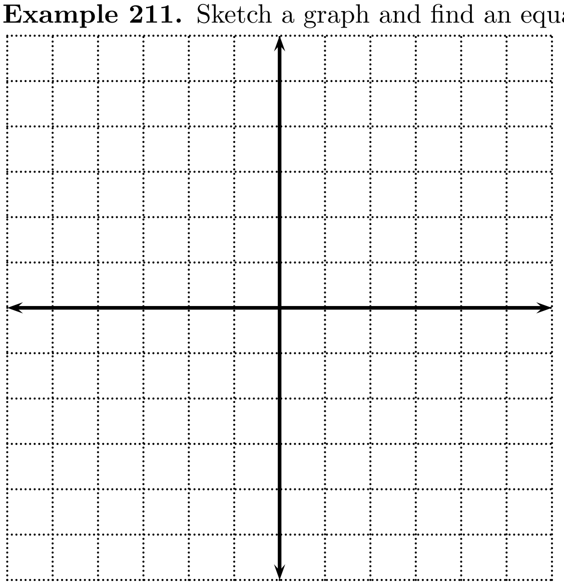
Example 209. Sketch a graph of $(y-4)^2 = -2(x+3)$. Find and label the vertex, axis of symmetry, focus and directrix.



Example 210. Sketch a graph and find an equation for the parabola with vertex $(2, -3)$ and focus $(4, -3)$.



Example 211. Sketch a graph and find an equation for the parabola with vertex $(1, 1)$ and focus $(1, -7)$.



General Form of a Conic:

Notes:

-

-

A and C tell us what kind of conic section it is:

-

-

-

Example 212. *(10.4: Try It #1)* Identify each of the following nondegenerate conic sections.

1. $16y^2 - x^2 + x - 4y - 9 = 0$

2. $16x^2 + 4y^2 + 16x + 49y - 81 = 0$

Chapter 8

Exponential Functions

8.1 4.1: Basic Exponential Functions and Models

The basic exponential function is:

- a:
- b:
- Sample graphs:
- End behavior:
- Domain:
- Range:
- Increasing or decreasing:
- Horizontal Asymptote:
- Vertical Intercept:

Important note: $f(x) = a \cdot b^{x-h} + k$ is also in the family of exponential functions

- h and k represent ____
- Vertical and horizontal shifts affect:
- $a < 0$ represents ________________________ and affects:

Identifying Exponential Functions

Example 213. *(Example 2 and Try It #1)* Which of the following are exponential functions? For the ones that are exponential functions, are they exponential growth or decay? For the ones that are not, what type of functions are they?

1. $f(x) = 4^{3(x-2)}$
2. $g(x) = x^3$
3. $h(x) = \left(\dfrac{1}{3}\right)^x$
4. $p(x) = (-2)^x$
5. $d(x) = 2x^2 - 3x + 1$
6. $j(x) = 0.875^x$
7. $k(x) = 1.75x + 2$
8. $q(x) = 1095.6^{-2x}$

Evaluating Exponential Functions

Example 214. Given $f(x) = 2(3)^{x-2}$, find $f(5)$.

Example 215. Given $f(x) = 8(1.2)^{x-5}$, use a calculator to find $f(3)$.

Basic Exponential Models

Example 216. *(Try It #2)* The population of China was about 1.39 billion in the year 2013, with and annual **growth rate** of about 0.6%.

1. Write a function $P(t)$ that models the population of China, where t is the number of years since 2013.

2. Use your model to predict what the approximate population of China will be in the year 2031.

3. *HW hint: This is similar to #61 in your HW.*

Example 217. *(HW #63)* In the 1985, a house was valued at \$110,000. By the year 2005, the value had appreciated to \$145,000.

1. Write a function $V(t)$ that models the value of the house, where t is the number of years since 1985, assuming that the value was growing exponentially.

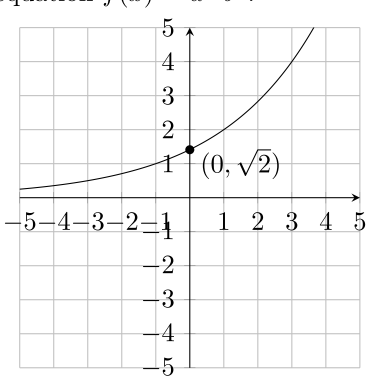
2. What is the annual **growth rate** of the value of the house?

3. Use your model to predict what the value of the house is today.

4. *HW hint: This is similar to #62 in your HW.*

Exponential Equations from Graphs

Example 218. *(Try It #6)* Write an equation for the function graphed below. Assume that it has a basic exponential equation $f(x) = a \cdot b^x$.



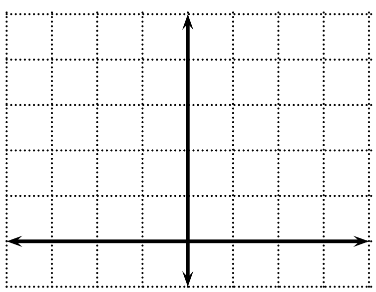
The Number e

Example 219. *(Try It #10)* Use a calculator to find $e^{-0.5}$. Round your answer to 5 decimal places.

8.2 4.2: Graphs of Exponential Functions

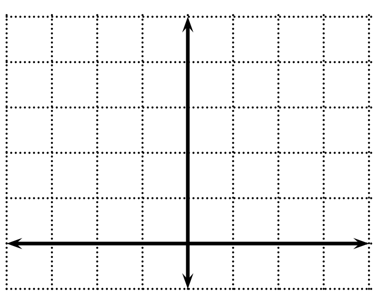
Example 220.
- Complete the table of values and use it to sketch a graph of $f(x) = 2^x$.

x	-2	-1	0	1	2
$f(x)$					



- Complete the table of values and use it to sketch a graph of $g(x) = \left(\frac{1}{2}\right)^x$.

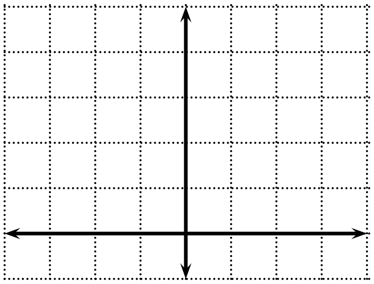
x	-2	-1	0	1	2
$f(x)$					



- Compare the graphs of $f(x)$ and $g(x)$

Example 221. *(Try It #1)* Find the following for $f(x) = 4^x$.

1. Sketch a graph.



2. Domain:

3. Range:

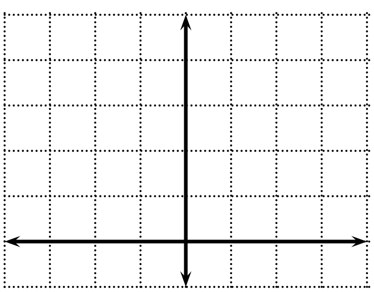
4. Growth or decay?

5. Horizontal Asymptote:

6. Vertical Intercept:

Example 222. *(Try It #4)* Find the following for $g(x) = \frac{1}{2}(4)^x$.

1. Sketch a graph.



2. Domain:

3. Range:

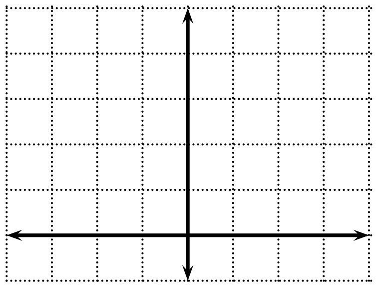
4. Growth or decay?

5. Horizontal Asymptote:

6. Vertical Intercept:

Example 223. *(Example 4)* Find the following for $h(x) = 4\left(\frac{1}{2}\right)^x$.

1. Sketch a graph.



2. Domain:

3. Range:

4. Growth or decay?

5. Horizontal Asymptote:

6. Vertical Intercept:

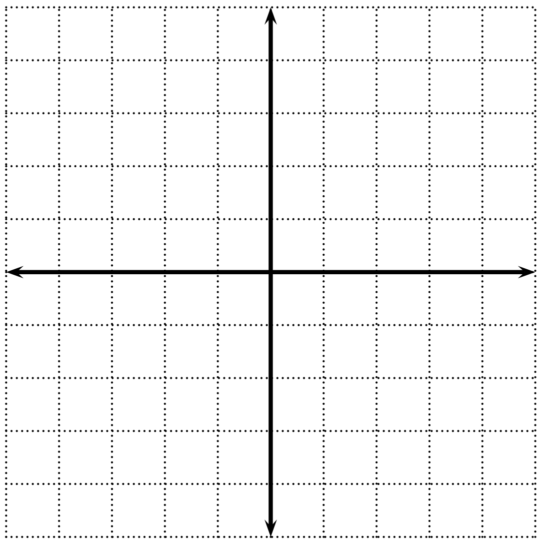
Compare the equations and graphs in Examples 221, 222 and 223.

Graph Transformations

Example 224. *(Try It #2)* Find the following for $f(x) = 2^{x-1} + 3$.

1. Parent function:

2. Use the parent function and graph transformations to sketch a graph of $f(x)$.



3. Domain:

4. Range:

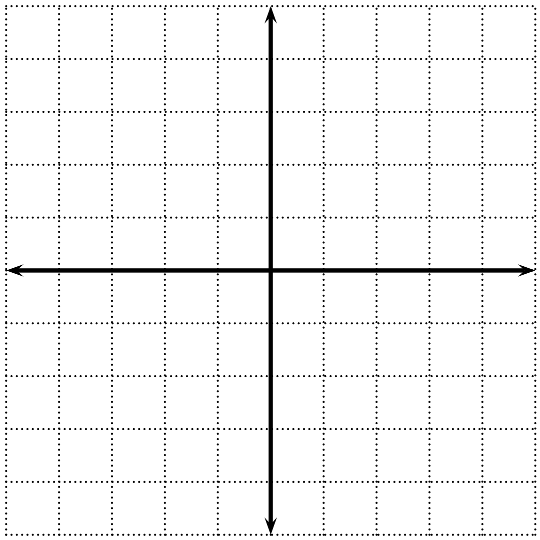
5. Horizontal Asymptote:

6. Vertical Intercept:

Example 225. Find the following for $f(x) = 3\left(\frac{1}{2}\right)^x - 2$.

1. Parent function:

2. Use the parent function and graph transformations to sketch a graph of $f(x)$.



3. Domain:

4. Range:

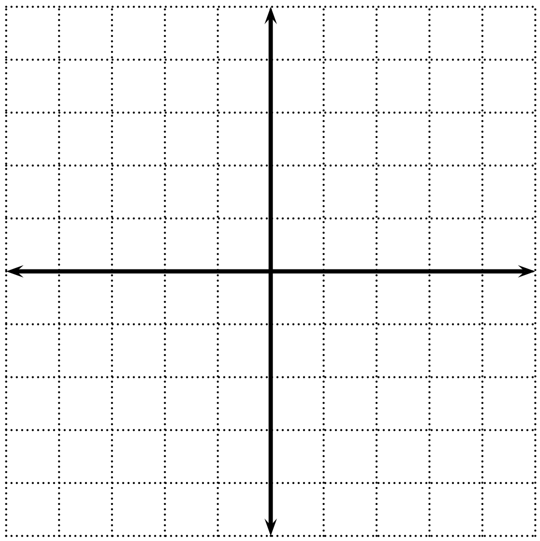
5. Horizontal Asymptote:

6. Vertical Intercept:

Example 226. Find the following for $f(x) = -(2)^{x+1} + 1$.

1. Parent function:

2. Use the parent function and graph transformations to sketch a graph of $f(x)$.



3. Domain:

4. Range:

5. Horizontal Asymptote:

6. Vertical Intercept:

Chapter 9

Logarithms

9.1 4.3: Intro to Logarithms

Example 227. Given $10^x = 500$, how could we solve for x?

What's the **inverse** of the basic exponential function $f(x) = b^x$?

Definition of the Logarithm

<table>
<tr><th colspan="3">Comparing Exponential and Logarithmic Functions</th></tr>
<tr><td></td><td>$f(x) = 2^x$</td><td>$g(x) = \log_2(x)$</td></tr>
<tr><td>Table of values:</td><td><table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$y = 2^x$</td><td></td><td></td><td></td><td></td><td></td></tr></table></td><td><table><tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$y = \log_2(x)$</td><td></td><td></td><td></td><td></td><td></td></tr></table></td></tr>
<tr><td>Graph:</td><td></td><td></td></tr>
<tr><td>Domain:</td><td></td><td></td></tr>
<tr><td>Range:</td><td></td><td></td></tr>
<tr><td>Asymptote:</td><td></td><td></td></tr>
<tr><td>Intercept:</td><td></td><td></td></tr>
</table>

Example 228. *(Try It #1a)* Write $\log_{10}(1,000,000) = 6$ as an exponential equation.

Example 229. *(Try It #1b)* Write $\log_5(25) = 2$ as an exponential equation.

Example 230. *(Try It #2)* Write the following exponential equations are logarithmic equations.

1. $3^2 = 9$

2. $5^3 = 125$

3. $2^{-1} = \dfrac{1}{2}$

Example 231. Evaluate the following logarithms.

1. $\log_3(81) =$

2. *(Try It #3)* $\log_{121}(11) =$

3. *(Try It #4)* $\log_2\left(\dfrac{1}{32}\right) =$

Common Logarithm

Example 232. Evaluate $\log(1,000)$

Example 233. Evaluate $\log(0.01)$

GROUP WORK: PRACTICE WITH LOGARITHMS

Example 234. Evaluate the following logarithmic expressions *without* a calculator

1. $\log(10,000)$

2. $\log_6 1$

3. $\log_5\left(\frac{1}{125}\right)$

4. $\log(0.1)$

5. $\log_4 0$

Example 235. Rewrite the following exponential equations as logarithmic equations.

1. $2^{2x-1} = 8$

2. $\left(\frac{1}{2}\right)^x = 4$

3. $27 = 3^{2x+1}$

4. $\left(\frac{1}{3}\right)^{-x} = 81$

Logarithms on Your Calculator

Example 236. Evaluate the following common logarithms using a calculator. Round your answers to 4 decimal places.

1. $\log(275)$

2. $\log\left(\frac{1}{2}\right)$

3. $\log\left(-\frac{1}{2}\right)$

Natural Logarithm

Example 237. Use a calculator to evaluate the following. Round your answers to 4 decimal places.

1. $\ln(0.01)$

2. $\ln(\sqrt{3}-2)$

3. $\ln(e)$

Back to Example 227:

Logarithms as Inverses

Example 238. Evaluate the following logarithmic expressions *without* a calculator

1. $\log_2 \left(2^{1.6}\right)$

2. $e^{\ln 6}$

3. $\log \left(10^2\right)$

4. $\log \left(100^2\right)$

9.2 4.4: Graphs of Logarithmic Functions

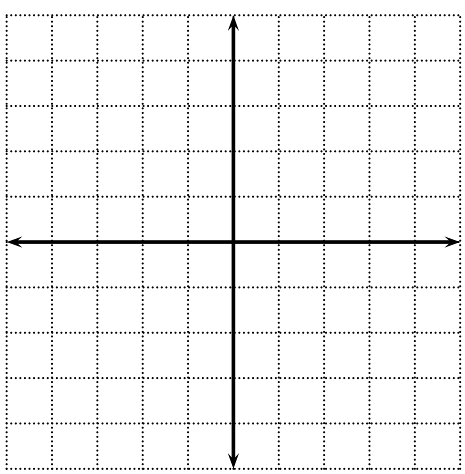
Exponential and Logarithmic Functions with $b > 1$		
	$f(x) = b^x$ $(b > 1)$	$g(x) = \log_b(x)$ $(b > 1)$
Graph:		
Domain:		
Range:		

Example 239. Find the following for $g(x) = \log_3(x)$

- Table of values:

x	-2	-1	0	1	2
$y = \log_3(x)$					

- Graph:



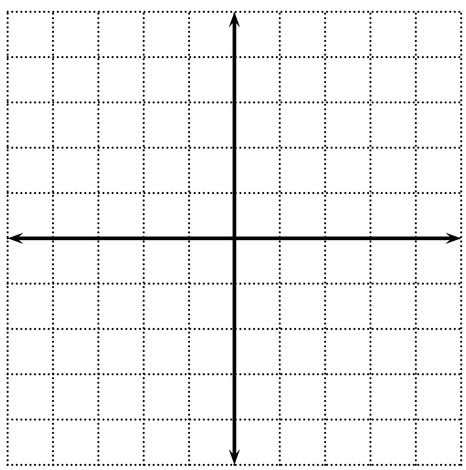
- Domain:
- Range:
- Vertical asymptote:
- Horizontal intercept:
- Increasing or decreasing?

Example 240. *(Try It #3)* Find the following for $h(x) = \log_{1/5}(x)$

- Table of values:

x	-2	-1	0	1	2
$y = \log_3(x)$					

- Graph:



- Domain:

- Range:

- Vertical asymptote:

- Horizontal intercept:

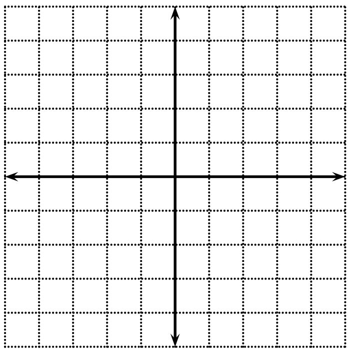
- Increasing or decreasing?

Comparing Logarithmic Functions		
	$g(x) = \log_b(x)$ $b > 1$	$h(x) = \log_b(x)$ $0 < b < 1$
Graph:		
Domain:		
Range:		
Vertical asymptote:		
Intercept:		
$f(b) =$		
Increasing or decreasing?		

GROUP WORK: GRAPHING EXPONENTIAL AND LOG FUNCTIONS

Example 241. Find the following for $f(x) = \log_{1/2}(x)$

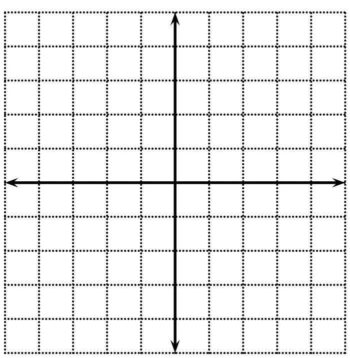
- Sketch a graph, including any asymptotes and at least 2 points, including any intercepts.



- Domain:
- Range:
- Asymptote:
- Horizontal intercept:
- Vertical intercept:
- Increasing or decreasing?

Example 242. Find the following for $g(x) = -4\left(\frac{1}{8}\right)^x$

- Sketch a graph, including any asymptotes and at least 2 points, including any intercepts.

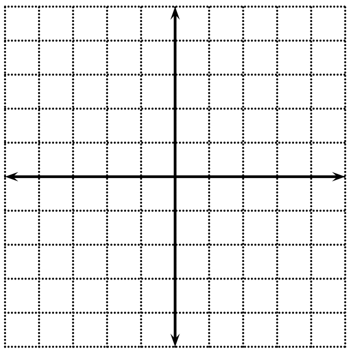


- Domain:
- Range:
- Asymptote:
- Horizontal intercept:
- Vertical intercept:
- Increasing or decreasing?

GROUP WORK: GRAPHING EXPONENTIAL AND LOG FUNCTIONS

Example 243. Find the following for $h(x) = \log_4(x)$

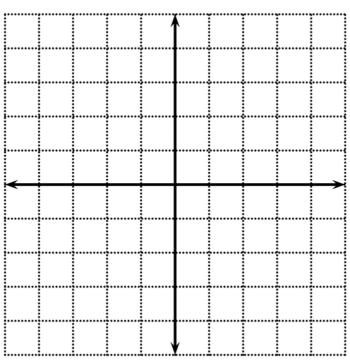
- Sketch a graph, including any asymptotes and at least 2 points, including any intercepts.



- Domain:
- Range:
- Asymptote:
- Horizontal intercept:
- Vertical intercept:
- Increasing or decreasing?

Example 244. Find the following for $p(x) = 2^x - 4$

- Sketch a graph, including any asymptotes and at least 2 points, including any intercepts.

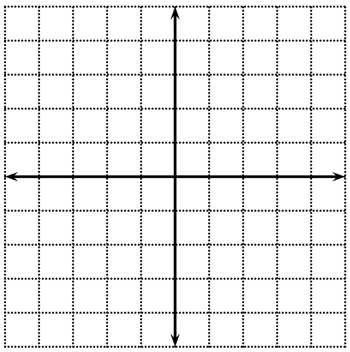


- Domain:
- Range:
- Asymptote:
- Horizontal intercept:
- Vertical intercept:
- Increasing or decreasing?

GROUP WORK: GRAPHING EXPONENTIAL AND LOG FUNCTIONS

Example 245. Find the following for $f(x) = -(10)^x$

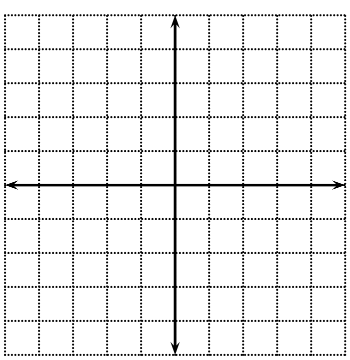
- Sketch a graph, including any asymptotes and at least 2 points, including any intercepts.



- Domain:
- Range:
- Asymptote:
- Horizontal intercept:
- Vertical intercept:
- Increasing or decreasing?

Example 246. Find the following for $g(x) = log_{0.3}(x)$

- Sketch a graph, including any asymptotes and at least 2 points, including any intercepts.



- Domain:
- Range:
- Asymptote:
- Horizontal intercept:
- Vertical intercept:
- Increasing or decreasing?

9.3 4.5A: Log Properties

Exponent Rules

- Product Rule
- Quotient Rule
- Power Rule

Product Rule for Logarithms

Example 247. *(Try It #1)* Expand $\log_3(8k)$

Example 248. Expand $\log_2(4x(x-1))$

Quotient Rule for Logarithms

Example 249. Expand $\log_5\left(\dfrac{25}{x+5}\right)$

Example 250. *(Try It #2)* Expand $\log_3\left(\dfrac{7x^2+21x}{7x(x-1)(x-2)}\right)$

Power Rule for Logarithms

Example 251. *(Try It #3)* Expand $\ln(x^2)$

Example 252. *(Try It #4)* Expand $\ln\left(\frac{1}{x^2}\right)$

Example 253. *(Try It #5)* Rewrite $2\log_3(4)$ as a single logarithm with coefficient 1.

GROUP WORK: PRACTICE WITH LOG PROPERTIES

Example 254. Expand each logarithm. State which property you use.

1. $\log_4(7xy)$

2. $\log_{16}\left(x^{1/4}\right)$

3. $\log_6\left(\frac{w}{z}\right)$

4. $\log_5\left(\dfrac{5}{x}\right)$

5. $\log_{12}(ab)$

6. $\log_5\left(x^5\right)$

Example 255. Condense each expression to a single logarithm with coefficient 1 and simplify further if possible. State which property you use.

1. $3\log_{11}(z)$

2. $\log_{11}(z) - \log_{11}(w)$

3. $3\log_8 7$

GROUP WORK: PRACTICE WITH LOG PROPERTIES

4. $\log_6 9 + \log_6 4$

5. $\log 4 + \log 3$

6. $\log_6 12 - log_6 2$

7. $\log_3 4 + \log_3 6 + \log_3 3$

8. $2 \log_{16} 6$

9. $3 \log(x + 1)$

10. $\log_4 \left(\sqrt[5]{4}\right)$

Example 256. Use log properties to evaluate the following without using a calculator.

1. $\log_5 \left(\sqrt[3]{5}\right)$

2. $\ln(e^6) - \ln(e^2)$

9.4 4.5B: Expanding and Condensing Logarithms

Expanding Logarithmic Expressions

Example 257. Full expand the following expressions.

1. *(Try It #6)* $\log\left(\frac{x^2y^3}{z^4}\right)$

2. *(Try It #7)* $\ln\left(\sqrt[3]{x^2}\right)$

3. $\ln(x^2 + y^2)$

4. *(Try It #8)* $\ln\left(\frac{\sqrt{(x-1)(2x+1)^2}}{x^2-9}\right)$

Condensing Logarithmic Expressions

Example 258. Condense each expression to a single logarithm with coefficient 1 and simplify further if possible.

1. *(Try It #9)* $\log 3 - \log 4 + \log 5 - \log 6$

2. *(Try It #10)* $\log 5 + 0.5\log(x) - \log(7x-1) + 3\log(x-1)$

3. *(Try It #11)* $4(3\log(x) + \log(x+5) - \log(2x+3))$

GROUP WORK: EXPANDING AND CONDENSING LOG EXPRESSIONS

Example 259. Fully expand the logarithmic expressions.

1. $\log_3 \left(\frac{4x^2}{\sqrt{y}} \right)$

2. $\log_2(16x^2y^4)$

3. $\ln \left(\frac{\sqrt{x+1}}{xy^2} \right)$

Example 260. Condense each expression into a single logarithm with leading coefficient 1. Simplify as much as possible.

1. $2(\log(x+3) - 2\log(x-2))$

2. $-4\log_6(2x)$

3. $\frac{1}{2}(\log_4(x+1) + 2\log_4(x-1)) + 6\log_4 x)$

Change of Base Formula

Example 261. Use your calculator to evaluate the following expressions. Round your answers to 4 decimal places.

1. *(Try It #13)* $\log_{0.5} 8$

2. *(Try It #14)* $\log_5 100$

3. $\log_2 12$

Chapter 10

Exponential and Logarithmic Equations

10.1 4.6A: Solving Exponential Equations

The one-to-one property of exponential equations:

Example 262. *(Try It #1)* Solve $5^{2x} = 5^{3x+2}$

Example 263. *(Try It #2)* Solve $5^{2x} = 25^{3x+2}$

Example 264. *(Try It #3)* Solve $5^x = \sqrt{5}$

Using Logs to Solve Exponential Equations

Example 265. *(Example 5)* Solve $5^{x+2} = 4^x$

Here's another way to solve this:

Example 266. *(Try It #5)* Solve $2^x = 3^{x+1}$

Example 267. Solve $2^x = 3^x$

Example 268. *(Try It #6)* Solve $3e^{0.5t} = 11$

This is a common technique use when solving equations:

1. **Isolate** the variable expression
2. **Undo** it to set the x free
3. **Solve** for x
4. **Check** your work

Example 269. *(Try It #7)* Solve $3 + e^{2t} = 7e^{2t}$

Exponential Equations as Quadratic Equations

Example 270. *(Try It #8)* Solve $e^{2x} - e^x = 56$

Example 271. *(Try It #9)* Solve $e^{2x} = e^x + 2$

10.2 4.6B: Solving Log Equations

Important: Always check for ____.

The one-to-one property of log equations:

Example 272. Solve each of the following equations. Be sure to check for **extraneous solutions**.

1. *(Try It #12)* $\ln(x^2) = \ln(1)$.
2. $\log_3 x = \log_3 12$
3. $\log(2x + 1) = \log(3x)$
4. $x = \log_{\sqrt{3}}(1)$
5. $x = \log_9 9$

Example 273. Solve for x. Be sure to check for extraneous solutions: $\log_5(x^2+3) = \log_5 12$

Using Exponents to Solve Log Equations

Example 274. *(Try It #9)* Solve $2\ln(6x) = 7$

Example 275. *(Try It #10)* Solve $2\ln(x+1) = 10$

Using Log Properties to Solve Log Equations

Example 276. *(HW #49)* Solve $\ln 3 - \ln(3 - 3x) = 3$

Example 277. *(HW #50)* Solve $\log_3(3x) - \log_3 6 = \log_3 77$

10.3 4.7: Applications

We can **model** exponential growth and decay using the equation

$$y = A_0 e^{kt}$$

where

- $A_0 =$

- $k =$

Half-life

Example 278. *(Try It # 14)*

- The half-life of Plutonium-244 is 80,000,000 years. Write an equation modeling the amount of Plutonium-244 remaining after t years. You may round k to 4 decimal places.

- Suppose we have 150 mg of plutonium-244. Use your model to predict when only 100 mg will remain.

Example 279. *(Try It # 15)*

- The half-life of Cesium-137 is about 30 years. If we begin with 200 mg of Cesium-137, will it take more or less than 230 years until only 1 mg remains? *(You may round k to 4 decimal places in your work.)*

Doubling Time

Example 280. A scientist is working with a culture of bacteria that doubles in size every 15 minutes. The initial population count was 1200 bacteria.

- Write an equation modeling the number of bacteria after t **hours**. You may round k to 4 decimal places.

- Use your model to predict the number of bacteria after 3 hours.

Example 281. *(HW #31-32)* A tumor is injected with 0.5 g of Iodine-125, which has a **decay rate** of 1.15% per day.

- Write an exponential model representing the amount of Iodine-125 remaining in the tumor after t days.

- Use your model to predict the amount of Iodine-125 that would remain in the tumor after 60 days. Round to 1 decimal place.

- To the nearest day, how long will it take for **half** of the Iodine-125 to decay?