

Instructions: Give complete solutions to the following problems be sure to provide all the necessary steps to support your answers.

1. Let  $A = PDP^{-1}$ , and compute  $A^5$

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

2. Which of the following matrices is diagonalizable. Prove your answer

$$\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

2. Find a matrix  $P$  that diagonalizes  $A$ , and compute  $P^{-1}AP$ .

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}, \lambda = 1, 5$$

3. Compute  $A^k$ .

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}, k = 5,$$

4. Consider the Matrix  $A$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$$

- Find a Basis for the Eigenspace of  $A$ , Call it  $\{u_1, u_2\}$
- Normalize the vectors in the Basis of the Eigen space of  $A$ . Call it  $\{q_1, q_2\}$
- Compute  $\lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T$
- Compare the answer to c with the Matrix  $A$ .