

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Compute the quadratic form $\mathbf{x}^T \mathbf{A} \mathbf{x}$, for

$$\begin{bmatrix} 3 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2. Find the matrix of the quadratic form. Assume \mathbf{x} is in \mathfrak{R}^n .

a. $3x_1^2 - 8x_1x_2 + 2x_2^2$

b. $2x_1^2 - 6x_1x_2$

c. $7x_1^2 + 4x_2^2 - 9x_3^2 - 8x_1x_2 + 6x_1x_3 - 108x_2x_3$

3. Make a change of variable, $x = Py$ that transforms the quadratic form $x_1^2 + 6x_1x_2 + x_2^2$ into a quadratic form with no cross-product term. Give P and the new quadratic form.

4. Make a change of variable, $x = Py$ that transforms the quadratic form $4x_1^2 + 4x_2^2 + 4x_3^2 + 4x_4^2 + 3x_1x_2 + 3x_3x_4 - 4x_1x_4 + 4x_2x_3$ into a quadratic form with no cross-product term. Give P and the new quadratic form.