Math02B Assignment 7.3 Last Name_______________1st_________

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Orthogonally diagonalize the given matrices, giving an orthogonal matrix P and a diagonal matrix D

a. $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 5 \\ 1 & 5 & 1 \\ 5 & 1 & 1 \end{bmatrix}. \lambda = -4, 4, 7$

2. Find a change of variable x = Py that transforms the quadratic form from $\mathbf{x}^T\mathbf{x}$ into $\mathbf{y}^T\mathbf{y}$.

$$Q(x) = 3x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3 = 5y_1^2 + 2y_2^2$$

3. Find (a) the maximum value of Q(**x**) subject to the constraint $\mathbf{x}^T\mathbf{x}=1$, (b) a unit vector **u** where this maximum is attained, and (c) the maximum of Q(**x**) subject to the constraints $\mathbf{x}^T\mathbf{x}=1$ and $\mathbf{x}^T\mathbf{u}=0$.

$$Q(x)=3x_1^2+3x_2^2+5x_3^2+6x_1x_2+2x_1x_3+2x_2x_3$$

4. Let $Q(x)=7x_1^2+x_2^2+7x_3^2-8x_1x_2-4x_1x_3-8x_2x_3$
which Q(**x**) is maximized, subject to $\mathbf{x}^T\mathbf{x}=1$.
[*Hint:* The eigenvalues of the matrix of the quadratic form Q are 9 and – 3.]