Acceleration in 1-D

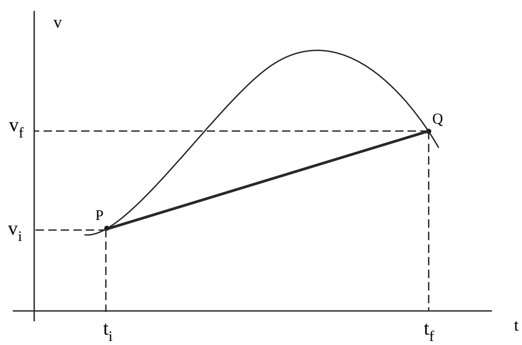
Acceleration – A measure of the time-rate of change of velocity

From a practical viewpoint we need to know how the velocity of a particle changes. The time-rate of change of velocity is what we call acceleration. Anytime the velocity of a particle changes we say that the particle has experienced an acceleration.

$$a_{ave} = \frac{\textit{change in velocity}}{\text{time interval}}$$ **Average Acceleration**

$$a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$ **Average Acceleration**

Average acceleration also has a geometric interpretation. Consider a particle moving between two points P and Q.



The slope of the line joining points P and Q is given by

$$slope = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} = a_{ave}$$

Thus, we have for the geometric interpretation of average accelration,

1. $a_{ave} =$ *slope of the line between two points on a v vs. t curve*

Following similar results as for instantaneous velocity, we have for instantaneous acceleration:

2. $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv(t)}{dt}$$
$$a = \frac{d}{dt}\left(\frac{dx(t)}{dt}\right) = \frac{d^2x(t)}{dt^2}$$

3. $a =$ *slope of the tangent line to a v vs. t curve at any time 't'.*

<u>Units</u>

$[\Delta x]$ = m
[v] = m/s
[a] = m/s/s = m/s^2

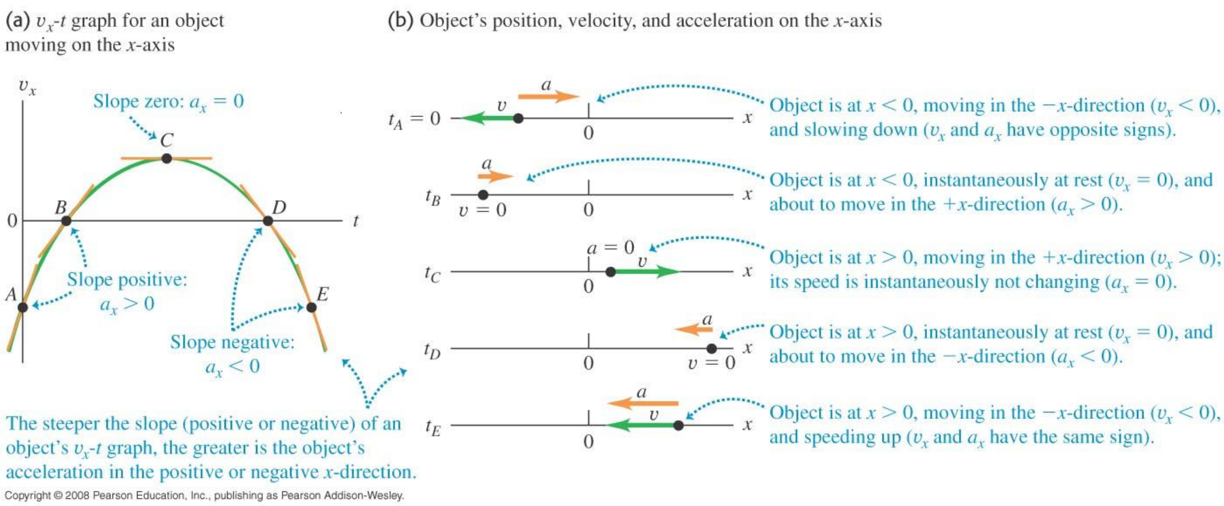
Ex. Consider a particle moving with constant acceleration of 2 m/s^2. Will the acceleration change? No – it's a constant. Since it's constant then a_{ave} = a. Thus,

a = $a_{ave} = \dfrac{\Delta v}{\Delta t}$ = 2 m/s^2 = 2 m/s/s = the velocity of the particle is changing by 2 m/s every 1 s.

For 1-D motion, acceleration implies speeding up or slowing down. However, the algebraic sign of acceleration doesn't tell you whether a particle is speeding up or slowing down. You must compare the algebraic signs of the velocity and acceleration.

a) If the velocity and acceleration have the same sign, the particle is speeding up.

b) If the velocity and acceleration have the opposite sign, the particle is slowing down.



Recall from calculus that if:

a) $\frac{d^2 f(x)}{dx^2} > 0$ on an open interval (a,b), then $f(x)$ is concave up on (a,b).

b) $\frac{d^2 f(x)}{dx^2} < 0$ on an open interval (a,b), then $f(x)$ is concave down on (a,b).

This implies that if:

a’) $a = \frac{d^2 x(t)}{dx^2} > 0$ on an open interval (a,b), then $x(t)$ is concave up on (a,b).

b’) $a = \frac{d^2 x(t)}{dx^2} < 0$ on an open interval (a,b), then $x(t)$ is concave down on (a,b).