Conductors in Electrostatic Equilibrium

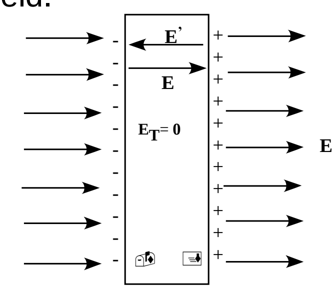
Electrostatic Equilibrium - When there is no net motion of charge within a conductor

A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on an isolated conductor must reside entirely on its surface.
3. The E-field just outside a charged conductor is perpendicular to the conductor's surface and has a magnitude $\frac{\sigma}{\varepsilon_o}$, where σ is the surface charge density at that point.

1st Property

The 1st property can be understood by considering a conducting slab placed in an external E-field.



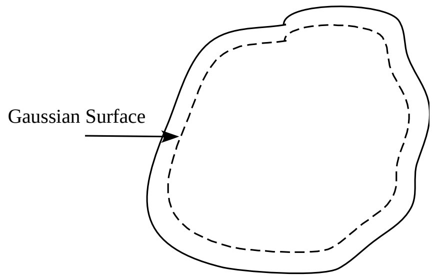
When the external E-field is applied:

a) the free e^- will accelerate to the left causing a negative surface charge density on the left and a positive surface charge density on the right.
b) These induced charges will produce and induced E-field that will oppose the external E-field.
c) Charge continues to accumulate until both E-fields cancel each other out inside the conductor.

The time it takes a conductor to reach this equilibrium is 10^{-16}s.

2nd Property

Consider arbitrary shaped conductor.



$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

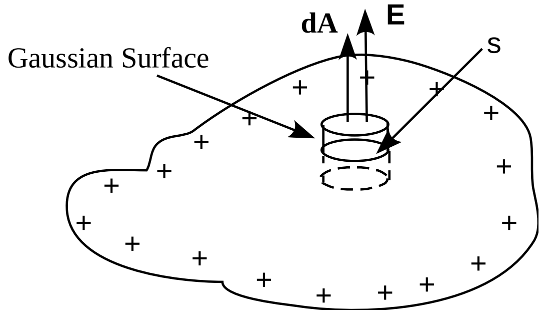
$$\vec{E} = 0$$

$$\therefore q_{enc} = 0$$

As we have just shown the E-field must be zero everywhere on the Gaussian Surface and thus the electric flux must also be zero. Therefore, the net charge inside the Gaussian Surface must also be zero. Because the Gaussian Surface can be made as arbitrarily close to the surface of conductor, the result still holds true and thus any net charge must reside on the surface of conductor.

3^{rd} *Property*

Once again consider a conductor. Construct a Gaussian surface in the shape of a small cylinder with the end faces parallel to the surface. Part of the cylinder is just outside the surface and the rest is inside.



Note that the E-field must be perpendicular to the surface. If it weren't it would have a component parallel to the surface and the charges would experience a net force and the conductor would not be in electrostatic equilibrium.

The net flux is due only to the top portion of the cylinder:

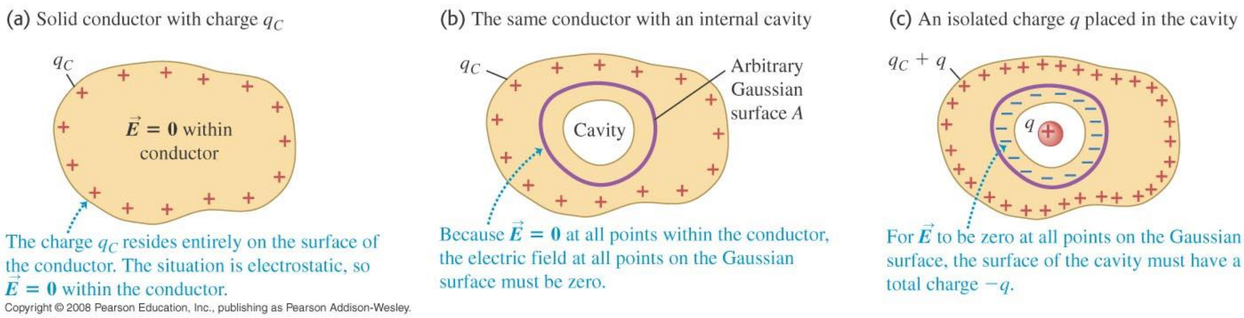
$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{q_{enc}}{\varepsilon_o}$$

$$EA = \frac{\sigma A}{\varepsilon_o}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_o}}$$ E-field at conductor surface

This result applies to any conductor in electrostatic equilibrium. It also shows that you have large E-fields where the surface charge density is large.



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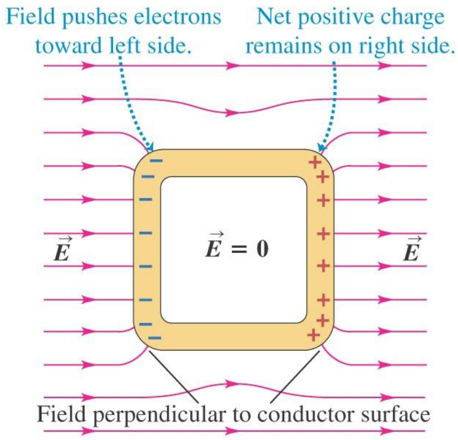
(a)



(b)

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