THE SIMPLE PENDULUM (Simple Harmonic Motion)

OBJECTIVE

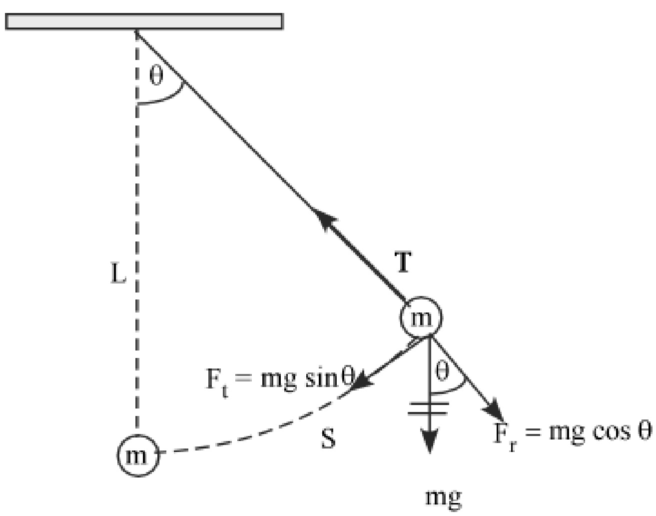
a) Calculate the acceleration of gravity ‘g’ and compare with expected value by analyzing the motion of a pendulum moving with Simple Harmonic Motion(SHM).
b) Calculate the length of a pendulum so that it can be used a pendulum clock.

EQUIPMENT

1. 2-m length of string
2. 1 large support rod, 1 small support rod, and 1 clamp
3. hanger
4. stopwatch
5. 2-m stick

THEORY

Consider a pendulum of length ‘L’ and mass ‘m’. Suppose the pendulum is swinging and at an instant in time its angular position is ‘θ’ with respect to the vertical. The Free-Body diagram for the pendulum is shown below at this instant in time.



1. Show that by applying N2L in the tangential direction ($\Sigma F_t = ma_t$) and by assuming small oscillations (small θ), the following equation must be satisfied:

$$\boxed{\frac{d^2\theta}{dt^2} + \left(\frac{g}{L}\right)\theta = 0} \text{ Simple Harmonic Equation}$$

2. Confirm that the solution to this equation is given by:

$$\boxed{\theta(t) = \theta_m \cos(\omega t + \phi)} \text{ Solution to SHM Equation}$$

Where,

$\theta(t)$ = amplitude of oscillation (rad)
θ_m = maximum amplitude of oscillations from equilibrium (rad)
$\omega = \sqrt{\frac{g}{L}}$ (angular frequency in units of rad/s) It is a measure of how fast the oscillations occur.
t = time (s)
φ = phase angle (determined by initial conditions) (rad)

3. The cosine and sin function repeat every period T. Thus:

$$\theta(t) = \theta(t+T)$$
$$\theta_m \cos(\omega t + \phi) = \theta_m \cos[\omega(t+T) + \phi)]$$
$$\theta_m \cos(\omega t + \phi) = \theta_m \cos[(\omega t + \phi) + \omega T)]$$

The sine and cosine repeat when their phase changes by 2π. Thus,

$$\omega T = 2\pi$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}} = 2\pi\sqrt{\frac{L}{g}}$$
$$T^2 = \left(\frac{1}{g}\right) 4\pi^2 L$$

4. The graph of T^2 vs. $4\pi^2 L$ will give a straight line with the slope related to the acceleration of gravity 'g'.

<u>PROCEDURE</u>

1. Setup apparatus
2. Measure length of pendulum (for corresponding length) from pivot point to the center of mass of hanger.
3. Measure the time for 10 oscillations and calculate the period. Repeat for same length for a total of 3 runs.
4. Calculate the average period for the 3 runs.
5. Repeat steps (2) – (4) for the length measurements indicated on the table below and fill in the data.
6. Make a graph of $T_{ave}^2 vs.\ 4\pi^2 L$ using EXCEL and obtain the equation of the best curve-fit.
7. Calculate the acceleration of gravity from equation.

8. Find the expected value of gravity g_{exp} on the Internet and compare with experimental result.
9. Using your equation for the best curve-fit , calculate the length of your pendulum so that it can be used as a pendulum clock.
10. Set the length of your pendulum to that calculated on step (9) and measure the time for 30 oscillations.
11. Can your pendulum be used as a clock? Explain your answer.

DATA TABLE

L(cm)	t_1	T_1	t_2	T_2	t_3	T_3	T_{ave}	T_{ave}^2	$4\pi^2$ L
50									
70									
90									
110									
130									
150									